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IMPROVED MODELS OF THE INNER AND OUTER RADIATION BELTS

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A pitch angle dependent invariant routine has been developed. This routine written in FORTRAN calculates the first and second invariant for an arbitrary number of pitch angles at any satellite location within the magnetosphere. The routine uses a new fast version of the IGRF internal magnetic field and the Olson-Pfitzer 1977 external magnetic field routine. The internal routine uses term dropping at large distances as well as improved coding to obtain speed advantages of 1.5 to 35 over the standard internal.

A vector potential model of the Chapman-Ferraro currents was developed for the March 1991 event which was observed by the CRRES satellite. This model was used to study the induction electric field and the electric field's importance on particle acceleration during the March 1991 injection event. An analysis using this induction electric field, which was calculated to be as large as .4 V/m, showed that it is capable of large increases in particle energy, but the induction electric field alone is not sufficient to explain the newly created inner belt. Greatly reduced cosmic ray cutoffs during the solar proton event which was in progress during the event, along with the induction electric field appear to be more probable source of the new bel

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1.0 Introduction

Considerable progress has been made in the understanding of magnetospheric processes in the last 25 years. During this time, much of the effort has been focused on understanding processes operating in the tail of the magnetosphere and near the magnetospheric boundaries. The inner magnetosphere has not been extensively investigated in the last 10 to 20 years. The Combined Release and Radiation Effects Satellite (CRRES) Program was designed to make a substantial contribution to understanding this region of space.

In this effort, McDonnell Douglas Space Systems Company (MDSSC) introduced novel new modeling approaches for modeling the inner magnetosphere and extensively studied a very large injection event that changed the character of the inner zone for many months. A number of tools were developed that have become valuable research tools. Most important, is a new B,L code that is pitch angle dependent and can be used with dynamic magnetic field models. Such models are an important tool for engineers for the design of systems that can survive in the space environment. The radiation belt models that are now used by both the scientific and engineering communities are all based on the Vette radiation models developed by the National Space Sciences Data Center (NSSDC) in the late 1960's and early 70's. The data sets which were used to develop the Vette models were acquired by instruments that are quite primitive compared to today's state-of-the-art instruments. Nevertheless, these older models have served the community well.

The present NSSDC developed models are organized in B, L space, a coordinate system developed by C. E. McIlwain in 1961. This coordinate system has been virtually unchanged since that time. Improvements have come only in the form of improved computational techniques. Although the B, L system has proven useful in the inner zone, its use in the outer zone has not been as successful.

This CRRES analysis effort included the development of new tools for the organization of the data. This effort developed novel new techniques for organizing charged particle data in the inner and outer zone. In the inner zone we developed tools to create a model that not only takes into account the effect of the magnetic field in organizing the charged particles but also the effect of the solar cycle dependent atmosphere in shaping the low altitude region of the inner radiation belt. The modeling effort, however, needs data spanning a portion of the solar cycle. The shortened life of CRRES precluded a full development of an atmospheric model. The tools, however, have become an important tool for other studies (notably the Space Environment and Effects, SEE, program). For the outer zone we have provided a coordinate system that can correctly represent adiabatic changes in the radiation belt and fully takes into account drift shell splitting and yet represents the entire outer zone in terms of only two parameters, the first and second invariant for each observation and pitch angle. The unique opportunity presented by the large enhancement to the inner zone cause by the March 1991 solar proton event and magnetic storm, for the first time demonstrated that induction electric fields are an important part of the energization process in the radiation belt.

One of the more significant parts of this was the development of the required computer code for defining the new coordinate system. By its very nature the

development of the computer code could not be completely decoupled from the analysis of the data. The quality of the data and the various features found within the data stream dictated the ultimate development of the model coordinate system.

2.0 Pitch Angle Dependent Invariant Routine and B,L code

In this section, we describe the development of pitch angle dependent invariant routine and the development of a new method for calculating B,L. Extensive use of this code by a number of researchers and by applying it to several low latitude data set has shown the code to be extremely accurate and robust. It has an accuracy in L of at least 0.001 Re and has proven to very efficient computationally. Since the initial development of the code and its tests by a number of researchers several changes have been made to the code to correct small coding errors. No large error or errors in the basic algorithm have been discovered. It is believed that the code is now fully tested and operational.

Calculating B,L has always been a computationally expensive procedure and thus considerable effort was expended to produce a B,L code that is efficient and cost effective. The calculation of the second invariant requires the calculation of a line integral that makes many calls to the magnetic field subroutine. Thus, some effort was expended to optimize the IGRF routines.

2.1 A Fast Version of IGRF

Present versions of B,L use only the internal model of the magnetic field. The CRRES code must use both internal and external models of the magnetic field, since it must take into account drift shell splitting in the outer magnetosphere. Thus, one of the most important routines for saving computer time is the development of an internal and external magnetic field routine that optimizes computer speed. The Olson-Pfitzer 1977 tilt dependent model is such a routine. It, however, uses the Barraclough internal field routine and thus is not

appropriate for the CRRES effort. The IGRF routines using the modern field coefficients were obtained from the National Space Sciences Data Center (NSSDC). These routines are, however, considerably slower than the routine contained in the original version of the 1977 tilt dependent model. The main field routine contained in the 1977 model is derived from Joe Cain's SPHRC routine. This routine gains additional speed at the expense of some memory. Instead of using indexed loops it explicitly writes out the spherical harmonic expansion terms and thus all of the overhead required to keep track of the various indices is abolished. It is this author's opinion that this version of representing the main field is inherently 50% faster than any other representation. The version used in the 1977 tilt dependent model has the Gauss normalized Barraclough coefficients built into the model. It, furthermore, has a term dropping algorithm, that drops the higher order terms as distances increases. This results in a considerable savings in computer time.

The IGRF internal field model developed for the CRRES analysis begins with Joe Cain's spherical harmonic expansion. The new IGRF routine has been given the name SPIGRF (SPeed IGRF). Since the IGRF coefficients are for a tenth order expansion, the 11th order term was added to the code. Terms up the N=11 are now contained in SPIGRF (the Barraclough model only had 10 terms). The first time SPIGRF is called, it calls a routine called FLDCOF. FLDCOF reads the appropriate IGRF coefficient sets, interpolates to the epoch of interest and converts the Schmitt normalized IGRF coefficients to Gauss normalized coefficients. If during a subsequent call, the date has changed by more than 0.1 year, FLDCOF is once again called to update the coefficients to the new epoch. It is not computationally efficient to update the coefficients for minor changes in time. In fact, it might be appropriate for the CRRES mission to select a specific

date and use coefficients for the internal magnetic field that do not change with time. This is a programmatic decision and will have little or no impact on the science since CRRES operated for only one year.

The term dropping algorithm that is a part of SPIGRF is a smooth algorithm. The algorithm uses predetermined altitudes where specific terms are to be discontinued. The smooth term dropping is possible since the spherical harmonic expansion is a complete orthogonal set. Table 1 lists the altitudes and maximum number of terms that are used by SPIGRF.

Thus, for altitudes greater than 12 R_e only the dipole term is used. For altitudes less than 1.4 R_e all 11 terms are used. For altitudes between 1.4 and 1.6, the contribution of the N=11 term is linearly reduced from its full value at 1.4 to zero at 1.6. Similarly for all other intervals. The altitude values at which the term dropping takes place was experimentally determined by comparing the truncated model with the untruncated version. The truncated model differs from the untruncated model by no more than 0.1 nanotesla.

This new IGRF code is inherently 1.5 times as fast as the NSSDC code, FELDG, when used with all the terms. The term drop off algorithm significantly improves the speed without sacrificing accuracy. Figure 1 presents the speed advantage of SPIGRF when compared to the original NSSDC code. At 3.0 R_e it is 3 times as fast as FELDG, 7 times as fast at 5 R_e, 10 times as fast at 7 R_e, and 35 times as fast at 12 R_e. Since the term drop off algorithm removes the effect of a term smoothly, there are no discontinuities in the field. For a typical CRRES orbit, this version of IGRF should have an average speed advantage of 7 or 8.

A listing and a brief description of the calling sequence for SPIGRF is given in Appendix A.

Table 1

Altitude, R_e	Number of terms
12.0	2
8.0	3
6.0	4
5.0	5
4.0	6
3.2	7
2.5	8
2.0	9
1.6	10
1.4	11

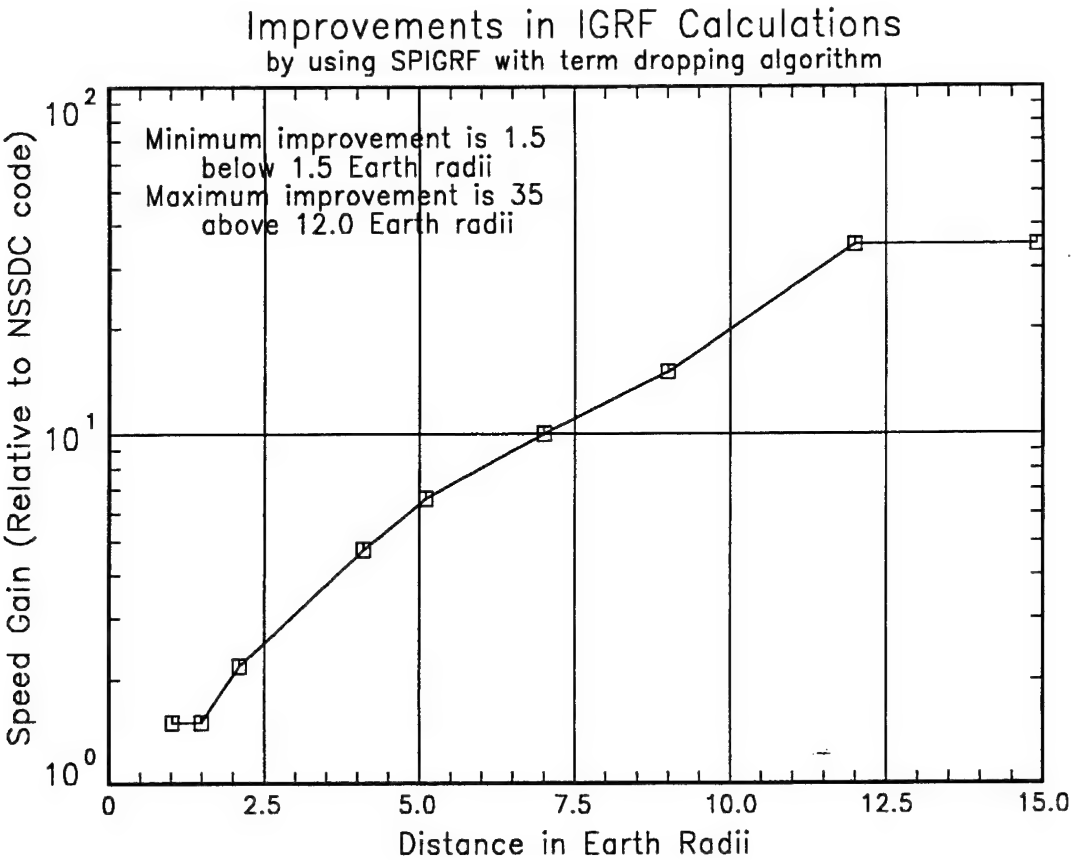


Figure 1. The improvement over the standard IGRF using the fast routine and the term drop off algorithm

2.2 Internal Plus the Tilt Dependent Quiet External Field

The new routine, SPIGRF, was combined with the remaining routines of the 1977 tilt dependent magnetic field model to produce a high speed quiet time magnetic field model that uses the IGRF coefficients. The name of the entire file that contains the tilt dependent model, SPIGRF, and a test routine was given the name BMNIGRF.

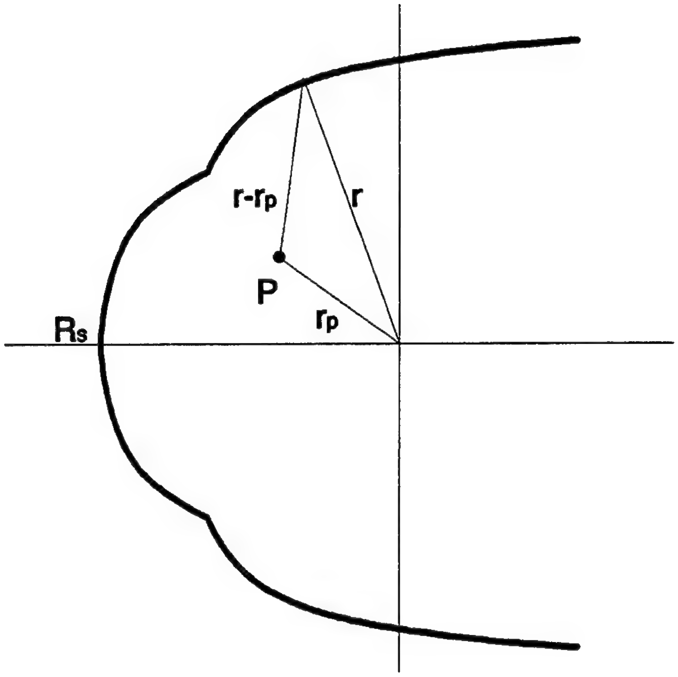
The execution speed of the new fast IGRF code, SPIGRF, plus the 1977 tilt dependent external model is faster everywhere than the old internal IGRF code, FELDG, without the external model. At 1.4 R_e or less the speed advantage is 1.2, at 2 R_e it is 1.05, at 6 R_e it is 1.6 and at 10 R_e it is 2.0. Thus, calculating B,L using the external and internal field routines is faster than calculating a B,L based on the internal field alone using the older IGRF routines.

Appendix B gives a listing of the BMNIGRF test routine, the 1977 tilt dependent routine, BXYZMU, and the various routines required to combine the external and internal magnetic fields. The routines in Appendix B when combined with the routines in Appendix A provide a complete internal plus external field model.

2.3 The External Field plus the Dynamic model of the External Magnetic Field

For out time dependent studies a time dependent external field must be used. The tilt dependent Olson Pfitzer model is a sum of magnetopause, ring and tail models combined into an easy to use single coefficient set model. For the dynamic model the various contributions are kept separate and are scaled to the appropriate algorithms.

2.3.1 Scaling Rules



The magnetic field at point P from the magnetopause currents when the stand-off distance is R_S is given by

$$\vec{B}(P) = \frac{\mu_o}{4\pi} \oint_{\substack{\text{magneto-}\\ \text{pause}}} \frac{\vec{\sigma}(x,y,z) \times (\vec{r} - \vec{r}_p)}{\left|\vec{r} - \vec{r}_p\right|^3} dA \quad (1)$$

where σ is the surface current system on the magnetopause. This surface integral performs the Biot Savart integral over the magnetopause. This equation is valid of any size magnetopause. We now wish to investigate the scaling rules for the magnetic field. We define a primed coordinate system where the magnetopause standoff distance is at 10.5 R_e. This is the quiet time position.

Then

$$\begin{aligned} x' &= S \cdot x \\ y' &= S \cdot y \\ z' &= S \cdot z \end{aligned} \quad (2)$$

where S is a scale factor and is equal to $10.5/R_S$.

This gives $\vec{r}' = S \cdot \vec{r}$ and $dA' = S^2 \cdot dA$

Thus, we can write

$$\vec{B}(P) = \frac{\mu_o}{4\pi} \oint \frac{\sigma\left(\frac{x'}{S}, \frac{y'}{S}, \frac{z'}{S}\right) \times \left(\vec{r}\,' - \vec{r}_p'\right) \cdot \left(\frac{1}{S}\right)}{\frac{1}{S^3}\left|\vec{r}\,' - \vec{r}_p'\right|^3} \frac{dA'}{S^2} \tag{3}$$

Let us assume that the surface currents scale as

$$\sigma\left(\frac{x'}{S}, \frac{y'}{S}, \frac{z'}{S}\right) = I_M(S) \cdot \sigma(x', y', z') \tag{4}$$

That is the currents in the scaled magnetopause have the same form as the unscaled differing only by a constant that depends on the scale factor. One thus has

$$\vec{B}(P) = I_M(S) \cdot \left[\frac{\mu_o}{4\pi} \oint_{\substack{magneto-\\pause}} \frac{\sigma(x', y', z') \times \left(\vec{r}\,' - \vec{r}_p'\right)}{\left|\vec{r}\,' - \vec{r}_p'\right|^3} dA' \right] \tag{5}$$

The expression in the brackets is the quiet time model evaluated at the scaled point r_p'. Thus, we can find the value of the magnetic field at point P when the magnetopause is compressed to R_S by evaluating the field at the a scaled point and multiplying by a scale factor that depends on the scaling parameter S, where $S = 10.5/R_S$.

In the quiet time model when $R_S = 10.5$, the magnetopause currents are defined such that the model magnetic field just outside the magnetopause everywhere cancels the dipole field. Thus, just inside the magnetopause the field is equal to the dipole field. Specifically, the field at the sub-solar point is given by

$$B_{10.5}(10.5, 0, 0) = \frac{M}{10.5^3} = 1.0 \cdot [\textit{Quiet Model}] \tag{6}$$

where M is the dipole moment. This says that on the sun-earth line at the sub-solar point just inside the boundary, the field from the boundary is equal to the dipole field.

When the magnetopause is compressed to a distance Rs, we have

$$B_{R_s}(R_S,0,0)=\frac{M}{R_S^3}=I_M(S)\cdot[\textit{Quiet Model}] \tag{7}$$

Using substitution on the above two equations we can show that

$$I_M(S)=\left[\frac{10.5}{R_S}\right]^3 \tag{8}$$

Thus, the magnetic field at point P in the compressed magnetosphere can be obtained by calculating the field in the quiet magnetosphere at scaled point P' and multiplying by scale factor $(10.5/R_S)^3$.

2.3.1 Determination of Standoff Distance

The key to the above model is the magnetopause standoff distance. In order to successfully determine the value of the magnetic field during disturbed times, a best guess must be made of the standoff distance. Thus, a substantial effort was used to obtain the standoff distances for the entire CRRES time period.

The standoff distance can be determined by several methods. Since the standoff distance is related to the solar wind dynamic pressure, if we know the density and velocity of the solar wind, the standoff distance can be determined by the relation $\boldsymbol{R_S=R_0 p^{-1/6}}$ where $\boldsymbol{p}$ is the solar wind dynamic pressure; $\boldsymbol{p=nV^2}$, $\boldsymbol{n}$ is the number density, and $\boldsymbol{V}$ is the velocity. The primary US spacecraft measuring the solar wind during the lifetime of CRRES was IMP-8. The "OMNI" file from NSSDC contains hourly averages of the IMP solar wind data, and is

easily accessible over the Internet; this was a prime source of data for determining the standoff distance in this study. Unfortunately, IMP-8 spends about one third of its time inside the magnetosphere, and telemetry coverage is not complete; therefore, solar wind data from this spacecraft are only available about 30-50% of the time. In addition, the IMP-8 data on the "OMNI" data file only covers the period up until about May 1991. Finally, the 1-hour time resolution is barely adequate for this study.

In order to augment the OMNI data, we have used an indirect technique to determine the standoff distance. Previous work has shown that, given a reasonably accurate value for the standoff distance (e.g., from solar wind data), magnetospheric models give excellent agreement with magnetometer data from spacecraft in geosynchronous orbit on the dayside of the magnetosphere [Olson and Pfitzer, 1982]. We, therefore, used geosynchronous magnetometer data, combined with the Olson-Pfitzer dynamic magnetic field model, to determine the standoff distance.

To do this, we used the magnetometer data from the GOES-6 and -7 spacecraft, which is available on floppy disk and CD-ROM from the National Geophysical Data Center (NGDC). The GOES data are provided as 5-minute averaged vector magnetic field values. These data were combined with D_{st} values from the OMNI file as input to the Olson-Pfitzer model. We then iterated on the standoff distance until the model B-north matched the measured B-north. This procedure was performed for both GOES-6 and GOES-7, which are at slightly different longitudes (about 135 and 100 degrees West, respectively), and thus at slightly different local times. To improve our confidence in the standoff distance calculations, we used data from a given spacecraft only when it was between 0900 and 1500 local time; in this region the Olson-Pfitzer model has been shown

to give excellent agreement with measurements. In order to obtain the best agreement between the two satellites, an offset of 10 nT was applied to the GOES-7 measurements; that is, we increased the measured value of B-north by 10 nT for input to the standoff distance algorithm. Finally, we found that the best agreement between the spacecraft and the available solar wind data was obtained when D_{st} was used to scale the strength of the ring current in the field model.

Figure 2 shows standoff distance over a five day period in August 1990, just after the launch of CRRES. The figure shows the standoff distance calculated from the GOES magnetometer data and from the IMP-8 solar wind data. Several points are noted. First, agreement between GOES-6 and GOES-7 is quite good most of the time, although there are periods when there is a considerable difference. Second, the two GOES spacecraft provide coverage for about eight hours a day; more complete coverage could be obtained if magnetometer data were available from other longitudes (e.g., from some of the LANL synchronous spacecraft). The agreement between the GOES standoff distance and that calculated from the solar wind data is also generally quite good. There are exceptions; for example, late on Day 227, the GOES data indicate a strong compression, below 7 R_E, while the solar wind indicates a standoff distance of about 8.6 R_E.

Figure 3 compares the standoff distance obtained from GOES-6 with that from GOES-7; as noted above, the agreement is quite good, especially at small standoff distances (although there are fewer points there). Figure 4 shows a histogram of the difference between the GOES-6 and GOES-7 standoff distances; most of the points are within ± 0.5 R_E, which is more than adequate for this study.

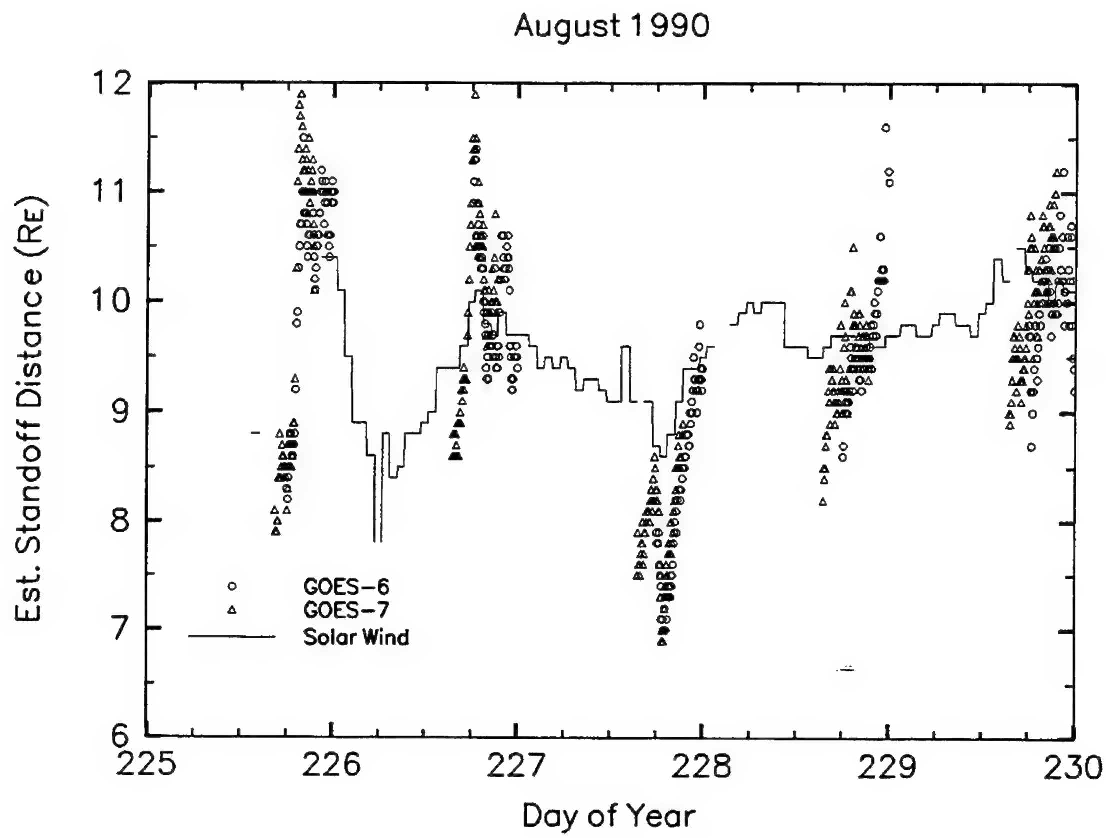


Figure 2. Time history of the magnetospheric standoff distance during August 1990, as determined by GOES-6 and -7 and the solar wind dynamic pressure.

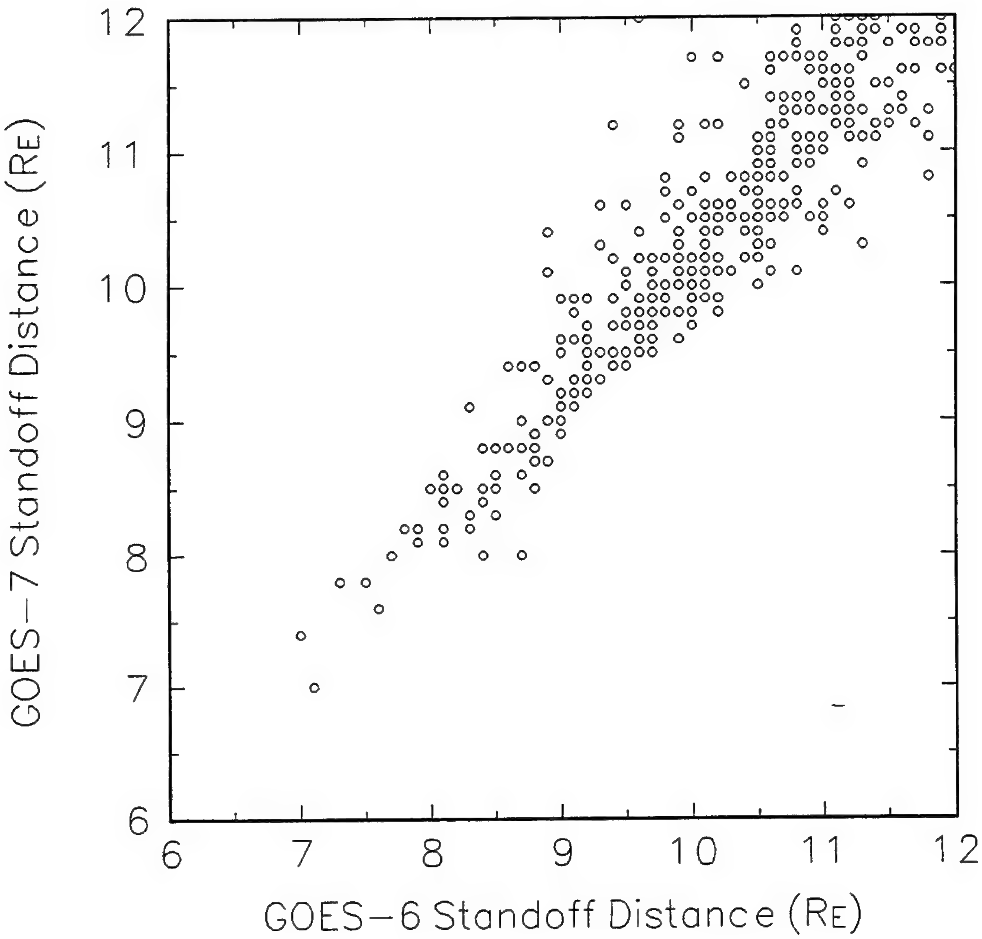


Figure 3. Comparison of standoff distance as determined by GOES-6 and -7.

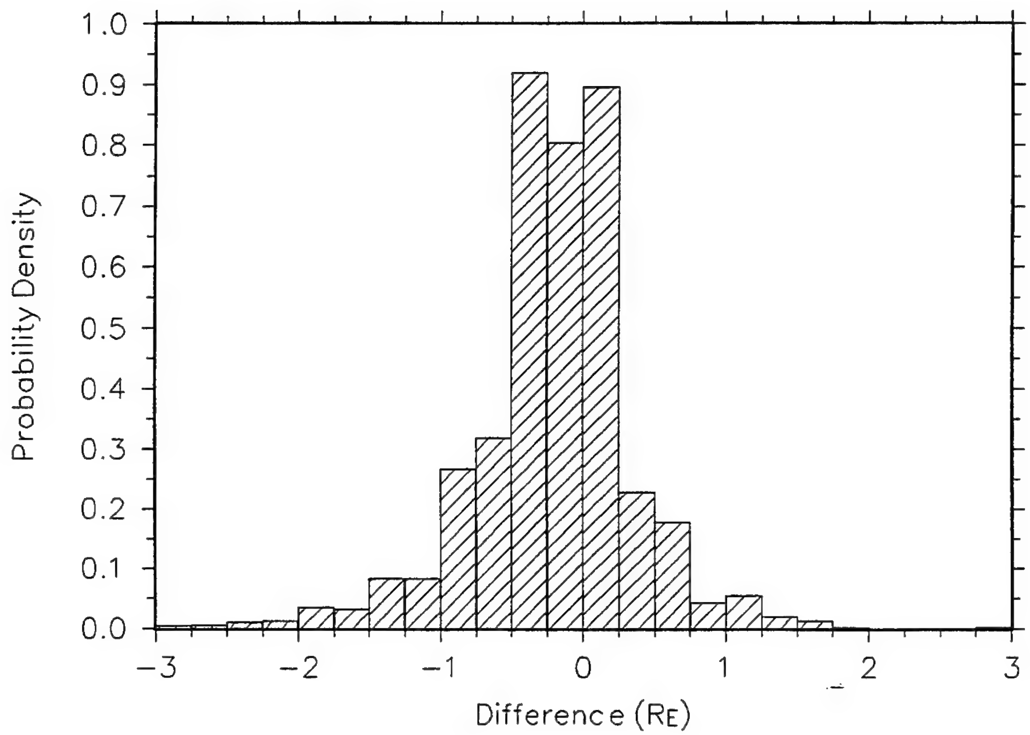


Figure 4. Histogram of differences between GOES-6 and GOES-7 satndoff distances

Figure 5 shows a histogram of the difference between the standoff distance obtained by GOES and that obtained from the solar wind data. The differences are larger, but still most of the points lie within ± 1R_E. One reason for the difference is that the solar wind data tend to be less variable, since they are 1-hour averages, compared to 5-minute averages. Also, the standoff distance is not a function of solar wind number density and velocity alone, but also depends on factors such as the interplanetary magnetic field and the ionic composition of the solar wind.

Finally, Figure 6 shows a histogram of the standoff distances obtained from the GOES data. The standoff distance is distributed more or less normally, with a mean of 10.4 and a standard deviation of 1.3 R_E. These are very close to the values of 10.1 and 1.4 R_E obtained by Petrinec *et al.* [1991], which increases our confidence in the technique.

A Personal Computer disk file was developed that contains the best guess standoff distances using the above algorithms. This file, in the form of a PKZIP file, contains ASCII data files containing magnetospheric standoff distance data for the period July 1990 - December 1991. There is one file for each month. Each file contains 7 columns of data, as described below.

DAY	The day of the year, including time of day, in decimal format
STD_6	The standoff distance as determined by the GOES-6 magnetometer
STD_7	The standoff distance as determined by the GOES-7 magnetometer
STD_KP	The standoff distance as determined by Kp
STD_PRESS	The standoff distance as determined by the solar wind dynamic pressure
DST	Dst (in nT)

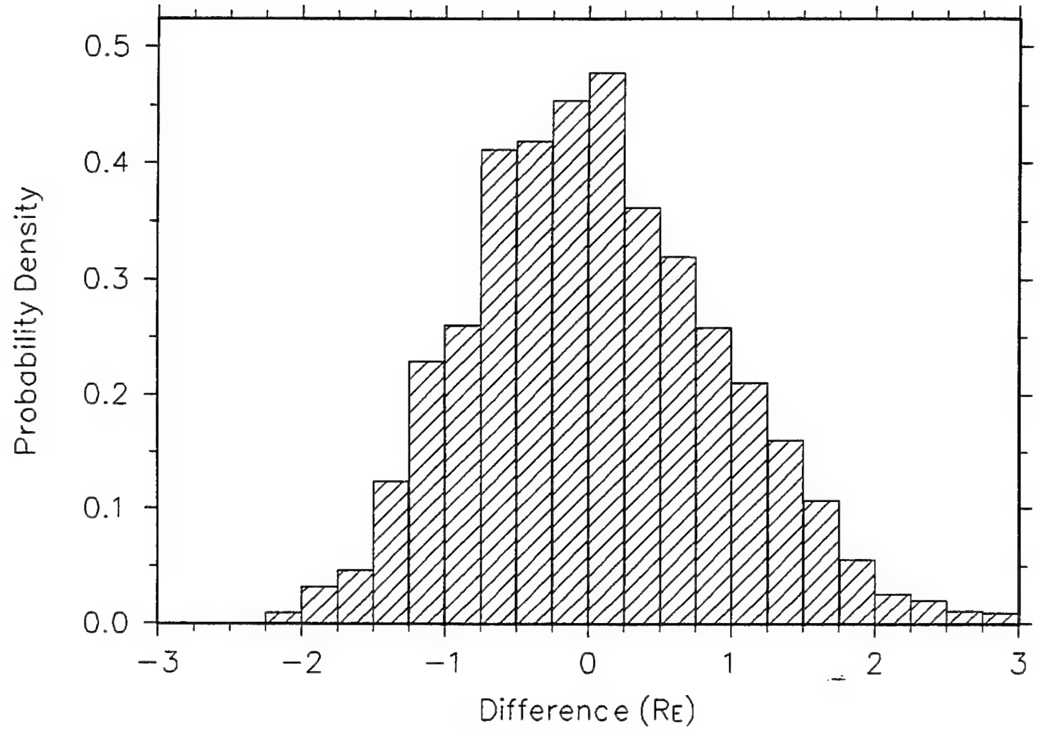


Figure 5. Histogram of the difference between the standoff distances determined by GOES-6 and -7.

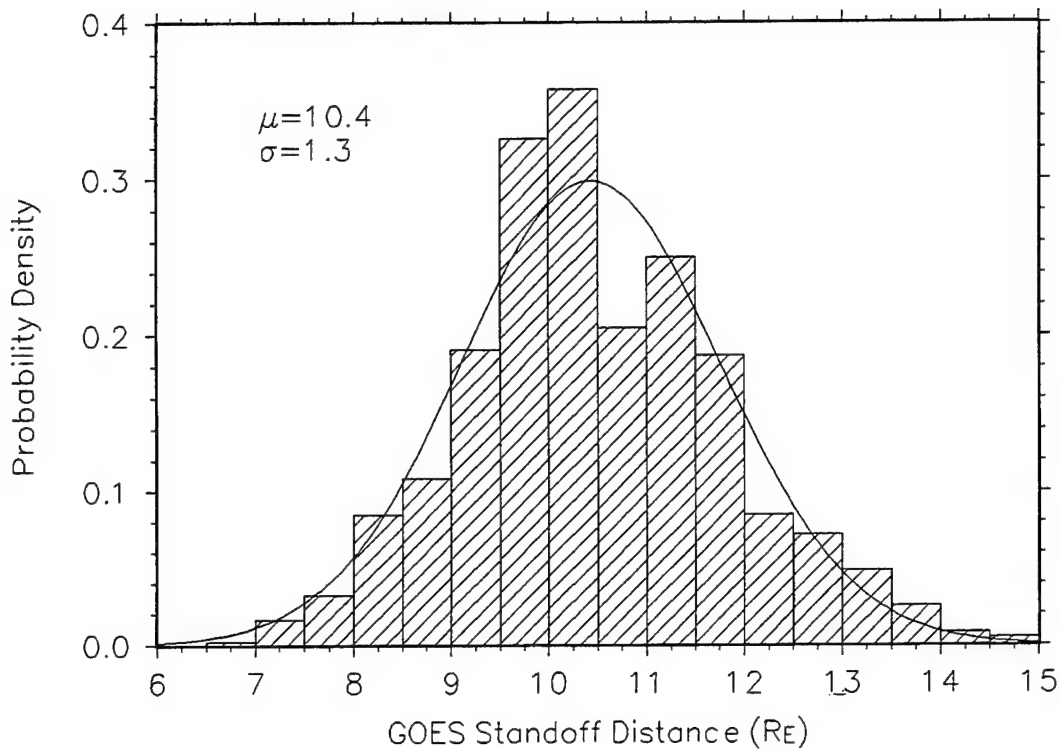


Figure 6. Histogram of the difference between standoff distances determined by GOES and the solar wind dynamic pressure.

BZ IMF Bz (in nT)

The first line of the file is a header record giving the names of the columns. The remaining lines contain the data written with the following format: (F10.4,4F5.1,F6.0,F5.1). The next few lines are a sample of the data.

```
day, std_6, std_7, std_kp, std_press, dst, bz
 182.0035   .0    .0 10.7 11.7    8.  -.5
 182.0069   .0    .0 10.7 11.7    8.  -.5
 182.0104   .0    .0 10.7 11.7    8.  -.5
 182.0139   .0    .0 10.7 11.7    8.  -.5
```

2.3.3 The Dynamic model

The Dynamic field model code described in Section 2.3.1 is listed and described in Appendix C. To convert the published B,L code to use the dynamic model, all of the routines listed in appendix B must be replaced with those listed in appendix C. Care must be used to set up the standoff distance, ring and tail parameters in the appropriate common block before calling the B,L code when using the dynamic model.

2.4 A Pitch Angle Dependent B,L Routine

Having defined a set of well-written efficient magnetic field routines, it is now possible to incorporate these magnetic field codes into a new pitch angle dependent B,L code. When used with the dynamic model, this B,L code can follow the adiabatic motion of the particles. The first and second invariant will track the changes of the magnetic field.

In order to adequately represent directional data, it is necessary to define the first and second invariant for each directional measurement. At the present time the current B,L routines calculate the invariant for a particle that mirrors at the

location of the satellite. All other particles will of course have different first and second invariant. The first invariant is simply the mirror point magnetic field of the particle and thus B_{mir} is given by

$$B_{mir} = \frac{B_{local}}{\sin^2(\alpha_{local})} \qquad (9$$

where B_{local} is the magnetic field at the location of the measurement and α_{local} is the local pitch angle of the measurement. Thus, the first invariant can be easily calculated for each directional measurement at a specific location.

The second, or integral invariant, J, is given by

$$J = \int_{B_{mir}}^{B_{mir}} \sqrt{1 - \frac{B}{B_{mir}}} ds \qquad (10)$$

This is a line integral over the bounce path of the particle and is calculated from one mirror point to the conjugate mirror point in the other hemisphere. If one needs to calculate the second invariant for more than one pitch angle, more than one integral must be evaluated. For a given line integral along a magnetic line of force between two conjugate mirror points, many calls must be made to the magnetic field subroutines. Since these routines contain spherical harmonic expansions or some other equally complex expansions for the internal field, the number of calls to the magnetic field routines must be minimized. If two particles have pitch angle α_1 and pitch angle α_2 and α_1 is greater than α_2, then the bounce path length of the pitch angle α_2 particle is longer than that of the α_1 particle. However, both particles will follow the same bounce path in the region of overlap. That is the line along which the integral is performed for pitch angle α_1 stops at B_{mir1} but that for pitch angle α_2 continues on through B_{mir1} to B_{mir2}. Unfortunately, since B_{mir} is inside the integral sign the value of the integral along the line differs for the two particles.

Maximum computer speed is obtained by dividing the invariant calculation into two parts. The first part calculates the path along the line of force and saves all of the pertinent parameters, and the second part calculates the integral invariant for each of the pitch angles. The multiple pitch angle invariant routine, INVARM, can calculate the first and second invariant of an unspecified number of pitch angles. The angles must be greater than zero (a pitch angle of zero would give an infinite first invariant) and less than or equal to 90 degrees. The pitch angle array must be sorted from biggest to smallest (i.e. 90,80,70,...). The line integral part of INVARM steps along a line of force with a step size that is dependent on the curvature of the line of force until the first B_{max} is reached. At each step in the integration the program calculates the step size as a function of the curvature of the field line. It also approximates from the present progress of the integration, the step size needed to reach B_{max}. It then chooses the smaller of the two steps. It attempts to get close to B_{max} without stepping past it on the first approach. It is important not to exceed B_{max} since the argument of the integral become imaginary if B_{max} is exceeded. The step size algorithm appears to work reasonably well and achieves an almost 100% success rate in not overstepping B_{max} on the first try. If B_{max} is exceeded the routine backs up and attempts to determine a step 'close' to B_{max} but smaller than B_{max}.

When the integration is first started, the routine first moves in the decreasing B direction in order that it can find the precise value and location of the minimum B. When minimum B is passed, the interpolation routine determines a precise value for B_{min} and also determines the magnetic longitude of minimum B. Once B_{max} has been found in one direction, the integral is re-started at the original location and that part of the line to the other mirror point is evaluated. Once the field line for the first pitch angle is found between the two mirror points, the

values stored by the field line code are used to evaluate the integral for the second invariant for the first pitch angle. To calculate the invariant for additional pitch angles, the field line portion of the routine is reentered and the line integration continues from the B_{max} stopping point of the previous pitch angle. The integration continues until the field line up to but not exceeding B_{max} of the next angle is determined. The integral for the second invariant is then calculated. This continues until the invariant of all of the pitch angles are determined or until one of the mirror points, either north or south is below 1.03 $R_{e,}$ or the maximum number of steps is exceeded, or until 13 R_e is exceeded. Each subsequent calculation utilizes all of the calculated values of the field strengths and step locations of the previous pitch angle, and thus the number of calls to the magnetic field line routines is minimized. For example, the computer time required to calculate the invariant for 18 pitch angles (90, 85, 80, 75,...,5) is approximate 2 to 3 times as long as the time required to calculate the invariant for the single pitch angle of 20 degrees.

The integration uses Gill's method of Runge-Kutta integration. This is a fourth order procedure and the error goes as step size to the fourth order. An internal error control parameter can be adjusted to control the errors. This parameter is set to give the "L" parameter an accuracy of at least 0.001. The maximum number of steps is 100. Since it is a fourth order procedure, up to 400 calls to the magnetic field routines are possible. Typically on the order 10 - 15 steps are required for a single pitch angle that mirror far off the equator (i.e. a pitch angle of 20 degrees at an L of 5.0). When the invariant for 18 pitch angles are calculated an additional step may be needed at both the northern and southern conjugate points for each additional pitch angle. If successive pitch angles are

very close together, the interpolation routine may be able to calculate the next invariant without the need of an additional step.

When the invariant routine calculates the second invariant it also integrates the total column density of the atmosphere between the mirror points. The density integral uses the atmospheric density function developed for the Air Force Office of Scientific Research. This function is given by

$$\text{density} = 2.7 \times 10^{-11} \exp[(120\text{-}z)/(\text{CON*sqrt}(z\text{-}103))] \qquad (11)$$

Where z is the altitude in kilometer and CON is an F10.7 dependent parameter (70 - 240) and is given by

$$\text{CON}= 0.99 + 0.518\text{* sqrt(F10.7/55)} \qquad (12)$$

Outside of 3.0 R_e or outside of a specified distance, the density function is arbitrarily set to zero, since the function has little validity above 1000 km altitude. It is, however, a smoothly decreasing function and can thus be used as an organizing parameter for atmospheric mirror depth up to 3.0 R_e.

Tables 2 - 5 are copies of the printouts for the calculation of the first and second invariant, and the 'L' parameter, and the density totals for a set of test conditions. Each page has two conditions. The top run uses the internal field only and the bottom run uses internal plus external field. All runs are started at latitude = 0, longitude =1.0 , Day of year = 1 and Universal time = 0. Table 2 is started at an altitude of 1.5 R_e. Both the internal and internal plus external runs give the same result since the external field is not important in this region of space. The small variations in L between the various pitch angles are due to the inaccuracies in the integration and more importantly to the accuracy and inherent approximate definition of the L expansion (see Hilton, J. Geophys. Res. 76, 6952, 1971). Pitch angles smaller than 35 degrees have their mirror point

Table 2

```
Lat =      .0  Long =      1.0  R =  1.5
Year =  1990.0  Day =      1.  UT =     .00  Field = INT
Blocal =    .08314  Bmin =   .08170  Mlat =     3.977  Mlong =    346.458
```

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.08314	.01980	1.548	3.49255E-17	82.45
85.0	.08377	.03084	1.549	7.59682E-17	80.95
80.0	.08572	.05440	1.548	6.75144E-17	77.49
75.0	.08911	.09770	1.547	1.13693E-16	73.25
70.0	.09415	.15871	1.547	2.15133E-16	68.68
65.0	.10121	.23789	1.547	4.75860E-16	63.96
60.0	.11085	.33591	1.546	1.42643E-15	59.15
55.0	.12390	.45367	1.545	5.81409E-15	54.30
50.0	.14167	.59234	1.545	4.52273E-14	49.41
45.0	.16627	.75347	1.544	7.56966E-13	44.51
40.0	.20121	.93914	1.544	4.50108E-11	39.58
35.0	.25270	1.15224	1.544	7.44210E-08	34.65
30.0	-1.00000	-1.00000	-1.000	-1.00000E+00	29.71
25.0	-1.00000	-1.00000	-1.000	-1.00000E+00	24.77
20.0	-1.00000	-1.00000	-1.000	-1.00000E+00	19.82
15.0	-1.00000	-1.00000	-1.000	-1.00000E+00	14.87
10.0	-1.00000	-1.00000	-1.000	-1.00000E+00	9.91
5.0	-1.00000	-1.00000	-1.000	-1.00000E+00	4.96

```
Lat =      .0  Long =      1.0  R =  1.5
Year =  1990.0  Day =      1.  UT =     .00  Field = IN+EX
Blocal =    .08314  Bmin =   .08170  Mlat =     3.977  Mlong =    346.458
```

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.08314	.01980	1.548	3.49255E-17	82.45
85.0	.08377	.03084	1.549	7.59682E-17	80.95
80.0	.08572	.05443	1.548	6.75032E-17	77.49
75.0	.08911	.09772	1.547	1.13681E-16	73.25
70.0	.09415	.15872	1.547	2.15143E-16	68.68
65.0	.10121	.23789	1.547	4.75860E-16	63.96
60.0	.11085	.33591	1.546	1.42642E-15	59.15
55.0	.12390	.45367	1.545	5.81420E-15	54.30
50.0	.14167	.59234	1.545	4.52266E-14	49.41
45.0	.16627	.75347	1.544	7.56985E-13	44.51
40.0	.20121	.93914	1.544	4.50081E-11	39.58
35.0	.25270	1.15224	1.544	7.44241E-08	34.65
30.0	-1.00000	-1.00000	-1.000	-1.00000E+00	29.71
25.0	-1.00000	-1.00000	-1.000	-1.00000E+00	24.77
20.0	-1.00000	-1.00000	-1.000	-1.00000E+00	19.82
15.0	-1.00000	-1.00000	-1.000	-1.00000E+00	14.87
10.0	-1.00000	-1.00000	-1.000	-1.00000E+00	9.91
5.0	-1.00000	-1.00000	-1.000	-1.00000E+00	4.96

Table 3

```
Lat =      .0  Long =      1.0  R =  3.5
Year =  1990.0  Day =      1.  UT =    .00  Field = INT
Blocal =    .00671  Bmin =   .00670  Mlat =     3.977  Mlong =    347.074
```

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00671	.00478	3.563	0.00000E+00	87.54
85.0	.00676	.02515	3.563	0.00000E+00	84.43
80.0	.00692	.08661	3.564	0.00000E+00	79.71
75.0	.00719	.18824	3.564	0.00000E+00	74.80
70.0	.00760	.33149	3.564	0.00000E+00	69.86
65.0	.00817	.51729	3.564	0.00000E+00	64.89
60.0	.00895	.74715	3.565	0.00000E+00	59.91
55.0	.01000	1.02299	3.565	0.00000E+00	54.92
50.0	.01143	1.34728	3.565	0.00000E+00	49.94
45.0	.01342	1.72315	3.565	3.61182E-31	44.95
40.0	.01624	2.15459	3.566	5.97057E-30	39.96
35.0	.02039	2.64682	3.566	1.14628E-28	34.96
30.0	.02684	3.20675	3.567	4.36298E-27	29.97
25.0	.03756	3.84408	3.568	4.41615E-25	24.98
20.0	.05735	4.57326	3.570	1.96738E-22	19.98
15.0	.10015	5.41832	3.574	1.23996E-18	14.99
10.0	.22250	6.42566	3.581	9.83762E-12	9.99
5.0	-1.00000	-1.00000	-1.000	-1.00000E+00	5.00

```
Lat =      .0  Long =      1.0  R =  3.5
Year =  1990.0  Day =      1.  UT =    .00  Field = IN+EX
Blocal =    .00629  Bmin =   .00628  Mlat =     3.977  Mlong =    347.076
```

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00629	.00323	3.641	0.00000E+00	87.86
85.0	.00633	.02177	3.640	0.00000E+00	84.56
80.0	.00648	.07700	3.637	0.00000E+00	79.78
75.0	.00674	.16928	3.632	0.00000E+00	74.85
70.0	.00712	.29965	3.625	0.00000E+00	69.89
65.0	.00765	.46972	3.617	0.00000E+00	64.91
60.0	.00838	.68166	3.607	0.00000E+00	59.93
55.0	.00937	.93820	3.597	0.00000E+00	54.94
50.0	.01071	1.24271	3.587	0.00000E+00	49.95
45.0	.01257	1.59937	3.577	7.74855E-32	44.96
40.0	.01521	2.01326	3.567	3.16436E-30	39.97
35.0	.01911	2.49064	3.559	5.99803E-29	34.97
30.0	.02514	3.03944	3.552	2.16502E-27	29.98
25.0	.03520	3.67102	3.547	2.05865E-25	24.98
20.0	.05374	4.39990	3.545	9.01949E-23	19.99
15.0	.09384	5.24888	3.545	5.15603E-19	14.99
10.0	.20847	6.26971	3.550	2.36695E-12	9.99
5.0	-1.00000	-1.00000	-1.000	-1.00000E+00	5.00

Table 4

Lat = .0 Long = 1.0 R = 5.5
Year = 1990.0 Day = 1. UT = .00 Field = INT
Blocal = .00176 Bmin = .00175 Mlat = 3.977 Mlong = 347.274

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00176	.02370	5.570	0.00000E+00	85.61
85.0	.00178	.05570	5.571	0.00000E+00	83.35
80.0	.00182	.15174	5.571	0.00000E+00	79.09
75.0	.00189	.31037	5.572	0.00000E+00	74.39
70.0	.00200	.53408	5.572	0.00000E+00	69.54
65.0	.00215	.82419	5.572	0.00000E+00	64.64
60.0	.00235	1.18302	5.572	0.00000E+00	59.71
55.0	.00263	1.61355	5.572	0.00000E+00	54.76
50.0	.00300	2.11971	5.573	0.00000E+00	49.80
45.0	.00353	2.70611	5.573	0.00000E+00	44.83
40.0	.00427	3.37914	5.573	0.00000E+00	39.86
35.0	.00536	4.14658	5.573	0.00000E+00	34.88
30.0	.00705	5.01889	5.574	0.00000E+00	29.90
25.0	.00987	6.01042	5.575	0.00000E+00	24.92
20.0	.01507	7.14198	5.575	0.00000E+00	19.94
15.0	.02632	8.44591	5.575	3.17060E-28	14.96
10.0	.05847	9.99589	5.578	4.08695E-23	9.97
5.0	.23210	11.96305	5.591	2.22143E-13	4.99

Lat = .0 Long = 1.0 R = 5.5
Year = 1990.0 Day = 1. UT = .00 Field = IN+EX
Blocal = .00150 Bmin = .00149 Mlat = 3.977 Mlong = 347.279

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00150	.00888	5.875	0.00000E+00	86.92
85.0	.00151	.03287	5.871	0.00000E+00	84.13
80.0	.00154	.10480	5.859	0.00000E+00	79.54
75.0	.00160	.22568	5.838	0.00000E+00	74.69
70.0	.00170	.39743	5.810	0.00000E+00	69.77
65.0	.00182	.62326	5.776	0.00000E+00	64.82
60.0	.00200	.90760	5.736	0.00000E+00	59.86
55.0	.00223	1.25613	5.693	0.00000E+00	54.88
50.0	.00255	1.67608	5.647	0.00000E+00	49.90
45.0	.00299	2.17641	5.602	0.00000E+00	44.92
40.0	.00362	2.76782	5.558	0.00000E+00	39.93
35.0	.00455	3.46351	5.518	0.00000E+00	34.94
30.0	.00599	4.27886	5.484	0.00000E+00	29.95
25.0	.00838	5.23284	5.455	0.00000E+00	24.96
20.0	.01280	6.34978	5.434	0.00000E+00	19.97
15.0	.02235	7.66456	5.418	4.39264E-29	14.98
10.0	.04965	9.24901	5.408	8.57689E-24	9.99
5.0	.19708	11.29810	5.417	2.64089E-14	4.99

Table 5

Lat = .0 Long = 1.0 R = 7.5
Year = 1990.0 Day = 1. UT = .00 Field = INT
Blocal = .00070 Bmin = .00070 Mlat = 3.977 Mlong = 347.369

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00070	.04747	7.576	0.00000E+00	84.68
85.0	.00071	.09120	7.577	0.00000E+00	82.71
80.0	.00072	.22136	7.578	0.00000E+00	78.69
75.0	.00075	.43727	7.578	0.00000E+00	74.11
70.0	.00080	.74148	7.579	0.00000E+00	69.33
65.0	.00086	1.13600	7.579	0.00000E+00	64.48
60.0	.00094	1.61743	7.576	0.00000E+00	59.58
55.0	.00105	2.20718	7.579	0.00000E+00	54.65
50.0	.00120	2.88932	7.576	0.00000E+00	49.71
45.0	.00141	3.68936	7.578	0.00000E+00	44.75
40.0	.00170	4.60022	7.577	0.00000E+00	39.79
35.0	.00214	5.64240	7.577	0.00000E+00	34.83
30.0	.00281	6.82682	7.577	0.00000E+00	29.86
25.0	.00393	8.17300	7.577	0.00000E+00	24.89
20.0	.00601	9.70984	7.578	0.00000E+00	19.91
15.0	.01049	11.48335	7.578	0.00000E+00	14.93
10.0	.02330	13.59271	7.583	1.81184E-29	9.96
5.0	.09250	16.22668	7.587	1.54025E-20	4.98

Lat = .0 Long = 1.0 R = 7.5
Year = 1990.0 Day = 1. UT = .00 Field = IN+EX
Blocal = .00056 Bmin = .00056 Mlat = 3.977 Mlong = 347.516

P. Angle	B mir	2nd Inv.	L	Density	Eq. Pitch Angle
90.0	.00056	.00053	8.130	0.00000E+00	89.31
85.0	.00057	.02219	8.119	0.00000E+00	84.95
80.0	.00058	.09740	8.091	0.00000E+00	79.98
75.0	.00060	.22835	8.047	0.00000E+00	74.98
70.0	.00064	.41436	7.985	0.00000E+00	69.99
65.0	.00069	.67362	7.915	0.00000E+00	64.99
60.0	.00075	1.00715	7.834	0.00000E+00	59.99
55.0	.00084	1.41840	7.744	0.00000E+00	54.99
50.0	.00096	1.93944	7.657	0.00000E+00	50.00
45.0	.00113	2.56532	7.568	0.00000E+00	45.00
40.0	.00136	3.33051	7.489	0.00000E+00	40.00
35.0	.00171	4.25138	7.420	0.00000E+00	35.00
30.0	.00226	5.35211	7.365	0.00000E+00	30.00
25.0	.00316	6.66066	7.326	0.00000E+00	25.00
20.0	.00482	8.21100	7.301	0.00000E+00	20.00
15.0	.00842	10.04846	7.289	0.00000E+00	15.00
10.0	.01870	12.27038	7.290	1.86371E-30	10.00
5.0	.07423	14.73075	7.168	7.07675E-22	5.00

below 200 km and the invariant is thus not evaluated. The internal field expansion diverges for distances less than 1.0 R_e, and thus mirror points below 1.0 cannot be assigned a second invariant. A -1.0 in any of the parameters indicates that the mirror point is too low in the atmosphere to calculate the invariant and assume that the particle is not trapped. A value of 100 indicates an open field line. As the altitude increases in Tables 3, 4 and 5, differences between the top run with internal field only and the internal plus external field run become increasingly large. The bottom run in Table 5, a run that calculates the invariant of a measurement at 7.5R_e, shows that shell splitting at 7.5 $R_e$is almost a full R_e.

The second to the last column in the printout shows the atmospheric density parameter. As the integration proceeds down the field line, the integration sums the atmospheric density producing a column number density for the amount of atmosphere a particle encounters as it bounces between the two mirror points. This number given in grams/cm^2 is intended to represent the importance of the atmosphere as a loss mechanism for the trapped particles. One observation can already be made from Table 2; 5 degree bins near the loss cone may not be an adequately fine resolution. As can be seen from Table 2, a 5 degree change in pitch angle (40 degrees to 35 degrees) causes the atmospheric column density to change by over two orders of magnitude. The next 5 degree bin mirrors below 200 km. Table 3 gives a 7 order of magnitude change in density encountered in the last 5 degree pitch angle bin. These numbers are an indicator on the expected sharpness of the atmospheric loss cone.

The last column displayed in the printouts shows the equatorial pitch angle for the specified particle at the present location. It is given by

$$\alpha_{eq} = \sin^{-1}\left[\sqrt{\frac{B_{min}}{B_{local}}}\ \sin(\alpha_{local})\right] \tag{13}$$

where α_{eq} is the equatorial pitch angle and α_{local} is the local pitch angle. It is important to remember that the equatorial pitch angle of a particle is not an invariant. The equatorial pitch angle of a particle changes as the particle drifts around the earth. This effect is most important at large distances where the earth's field becomes more asymmetric.

INVARM, efficiently calculates the first and second invariant of numerous pitch angles at a single satellite position. It also calculates the effective L's for the given set of invariant. It, furthermore, determines the actual minimum B field along the field line, the magnetic latitude of the observation point and the magnetic longitude where the field line crosses the magnetic equator. It also provides the column density of the atmosphere between the mirror points and thus is able to estimate the amount of scattering or absorption that can take place at the specified pitch angle. INVARM provides a complete characterization of all of the pertinent magnetic parameters for any set of pitch angles anywhere within the stable trapping region.

Appendix D lists routine INVARM, a test routine, and the various subroutines and functions required for its correct operation. The magnetic field routines required to define the field are described in Appendix A and B. Appendix C lists a dynamic magnetic field subroutine that can be substituted for the external magnetic field that is distributed with the fully tested version of INVARM. It allows for the calculation of dynamic L values as a function of magnetic conditions within the magnetosphere.

2.4 Enhancements to the B,L Routine for an Atmospheric Dependent Model.

The published and tested B,L code contains an algorithm that determines the integrated number density in gm/cm^2 along a magnetic line of force. Recent work and discussions indicate that this is more than likely not the correct parameter. There are several additional parameters that must be investigated. Of interest is the number of particles that the actual particle encounters on a drift shell and thus its probability of interaction with the atmosphere. This is a function of the path of the spiral path of the particle in the drift shell and not the path along the field line. Thus, a more accurate parameter would be an integral of the number density that takes into account the amount of time spent at each point along the field line and at each longitude. The time spent at each point along a field line is inversely proportional to the parallel velocity of the particle along the field line. Thus, the density integral should contain a $1/v_{par}$ term. This introduces a singularity into the integration of the number density along the particle path. It is, however, a singularity that can be integrated. The code for this option is working but has not been fully rung out and thus is not included in this report.

A second more basic design criteria in developing a model that reflects the changes of the atmosphere at low altitudes is that the atmospheric density along a particle's entire drift shell is most probably the best organizing parameter. Thus, when initially investigating how fluxes at low altitude depend on atmospheric density it is important to determine a drift shell density total or density average. This requires evaluating the density everywhere along the drift shell of the particle, a huge increase in computer time. We have developed an algorithm for mapping out the actual drift shell that we believe to be more

efficient than earlier algorithms. There is still a 20 to 50 fold increase in computer time. The B,L code as written determines the location of the magnetic minimum or magnetic equator. At the present time, only the latitude and magnetic longitude of this point is returned to the user. However, the actual Cartesian position of the magnetic minimum is saved in variable RMIN(3) in common block INTPAR. We have developed an algorithm for mapping out a drift shell, by first passing a circle through RMIN, having its center at the location of the offset dipole and its plane perpendicular to the offset dipole, and then stepping in longitude along this circle to a new starting location and reevaluation B,L. More often than not, we find that the L at the new location is within our 0.002 R_e of the specified L on the first try. If it is not, an adjustment in altitude which is based on the error in L always finds the correct field line on the second try. We have found that stepping in longitude with intervals of 15 to 20 degrees when not on field lines that dip into the South Atlantic Anomaly and reducing the spacing near the South Atlantic to 3 to 5 degrees, gives good and consistent results with a minimum of computer time. Computer time, however, is still excessive since on the order of 30 to 40 calls to the field line routine are necessary for each shell. Ultimately some kind of curve fit must be used for the final model. This will be determined by the actual atmospheric density dependent model. Early investigations using low altitude satellite data indicate a substantial hysteresis in the solar cycle effects and thus the form of a solar cycle dependent model is unknown at this time.

2.7 Putting It All Together, A Pitch Angle Dependent Representation.

The subroutine INVARM returns not only Bmax and L, it also returns the value of the second invariant. In order to organize all of the pitch angle dependent data, we have chosen to organize the data directly in terms of Bmax and J, the second

adiabatic invariant. The problem with Bmax and J is that these values do not give the user a feeling of geometry, the user has no idea of where in the magnetosphere the given J and Bmax is located. We have thus defined an L* where

$$L^* = \left[{M}/{B_{mir}} \right]^{1/3} \tag{14}$$

At the magnetic equator where J = 0, L* is effectively the dipole L expressed in Re. If we now define $J^* = J^{(1/3)}$, we find that the coordinate space looks very much like a geographic coordinate system. Figure 7 is an example of such a plot.

Figures 7 and 8 are plots of CRRES proton data during a small solar event. The inner zone protons are clearly visible out to an equivalent L of about 4.0. We also see a band of protons for L>5.8 out to about 6.5 the maximum distance for CRRES. The plot which has the color coded intensities plotted as a function of J* and L* also shows the lines of constant L and constant equatorial pitch angle. We note that this plot looks very much like the old L versus magnetic latitude style plots. It, however, is for pitch angle dependent data and the equatorial angle distribution at any given L shell can easily be determined for each location.

We used our solar cosmic ray trajectory program to determine the cutoff for this configuration. This is a time period when both the Dst and a measure of the standoff distance is available. Figure 9 gives the Dst and Figure 10 gives the standoff distance as a function of time during this small solar proton events. Initially, the standoff distance is large, during orbit 75 it varies from 9.5 to 10.5 and during the inbound orbit of orbit 76 before the sudden commencement the standoff distance is larger that 11.0. When we use a nominal standoff distance

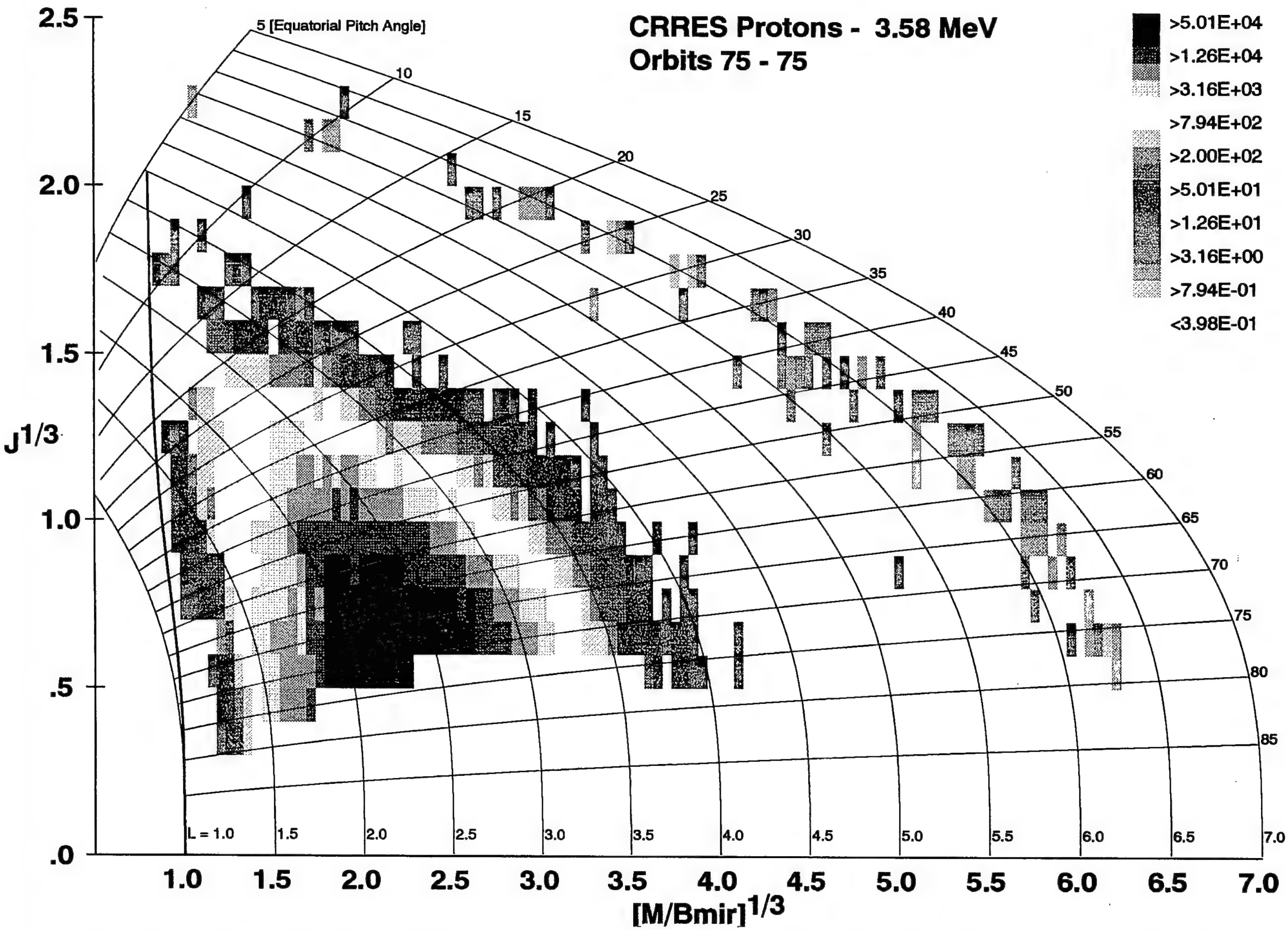


Figure 7. A plot of intensity in the first and second invariant space. Note inner zone protons and protons out at 6.0 Re due to a small solar proton events

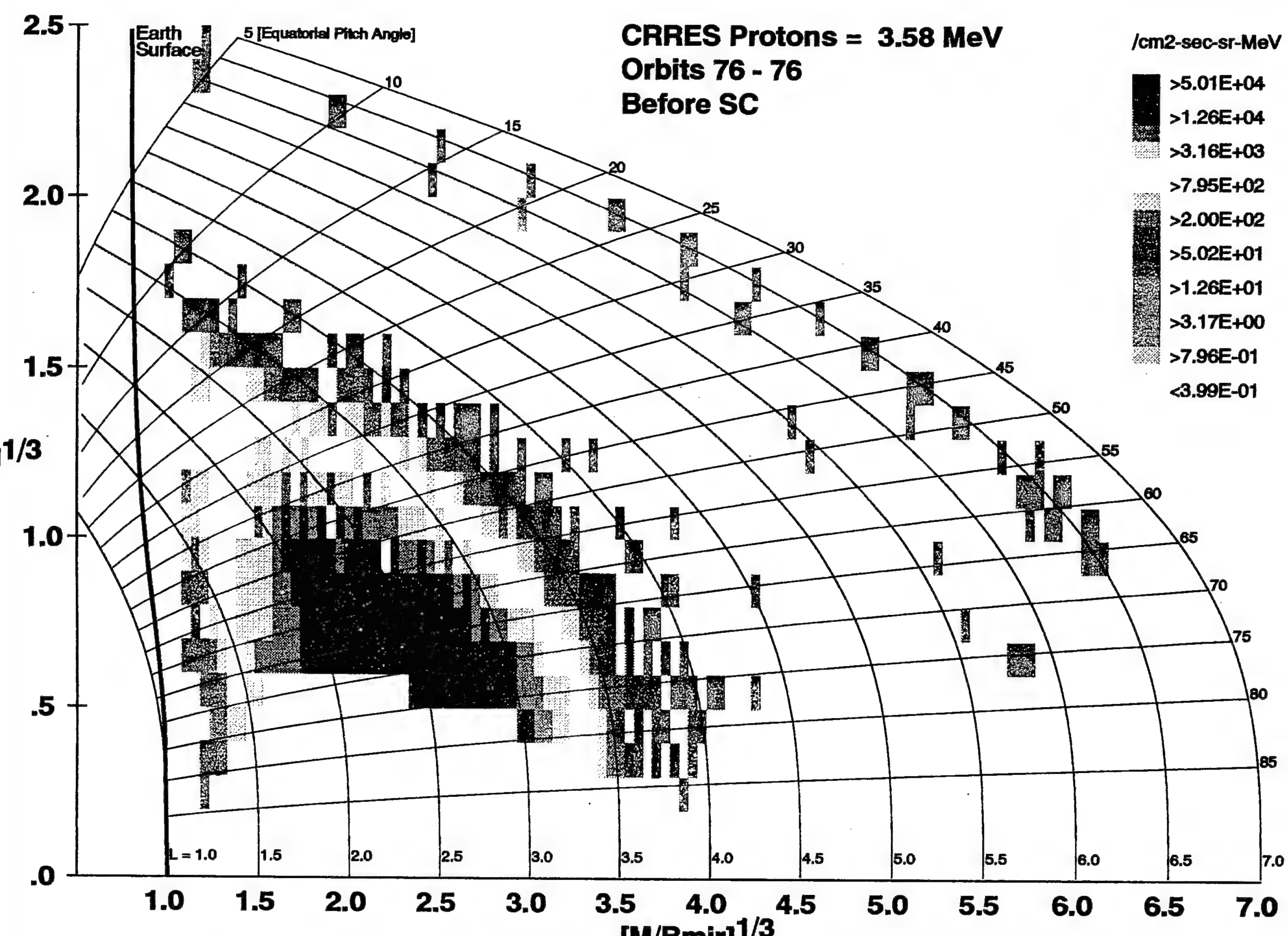


Figure 8. A plot of intensity in the first and second invariant space. Note inner zone protons and protons out at 6.0 Re due to a small solar proton events

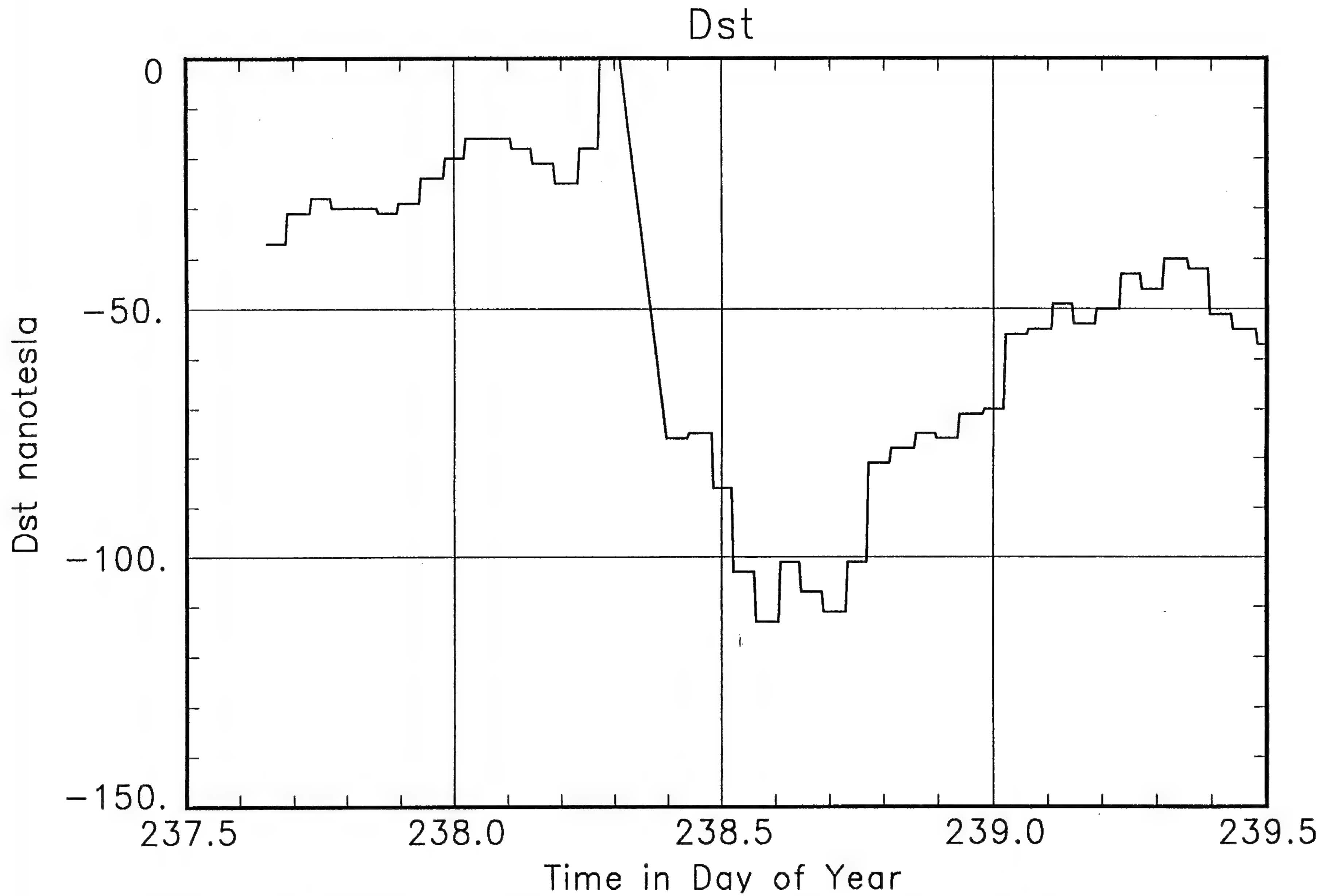


Figure 9. The Dst showing the sudden commencement followed by a buildup of the ring current.

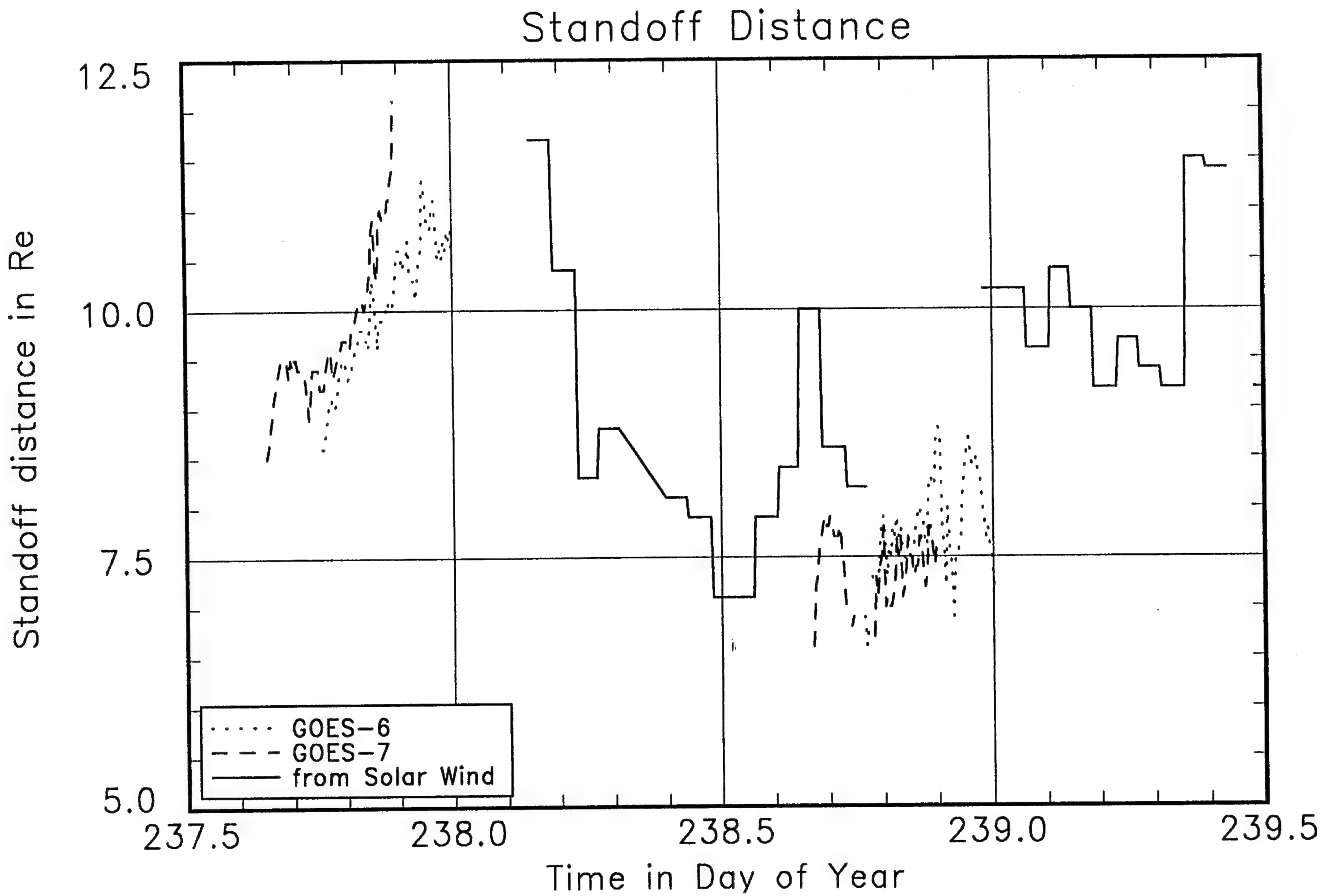


Figure 10. Best guess standoff distance from our standoff distance data base.

of 10.5 to calculate the cutoffs, we calculate a cutoff of close to 6.5. This is in reasonable agreement with a measured cutoff of about 5.8 on orbit 75 (Figure 7) and 6.3 on orbit 76 (Figure 8). Note, that it is possible to see the difference between the small changes in standoff distance before the sudden commencement. The cutoff program is only accurate for 30,000 to 50,000 integration steps. This allows the particle several circuits around a drift shell. Longer integration times would find the occasional particle that can indeed deterministically reach these lower cutoffs, but because of integration errors one can no longer trust the results.

From Figure 9 we see that there is a sudden commencement on day 238 at about 6 UT. At this time, the magnetopause is compressed to a standoff distance of about 7 (Figure 10) and remains in this compressed state until at least day 239. The second half of orbit 76 (Figure 11) and orbit 77 (Figure 12) where the magnetosphere is compressed, the cutoff is somewhere between 4 and 4.5. Calculation of the cutoff using a standoff distance of 7 gives a cutoff of about 5.5. The change in standoff distance lowers both the theoretical and measured cutoff by about 2 Re. On day 239, when the standoff distance has moved back out to 9.5 to 10 Re, the cutoff as seen in Figure 13 is once again on the order of 5.8.

The above exercise shows the value of the new coordinate system and demonstrates the sensitivity of the cosmic ray cutoffs to the magnetic configuration of the magnetosphere. It thus shows the importance of using a dynamic model in representing the fluxes in the outer zone. The dynamic model becomes even more important when attempting to understand the injection and motion of particles during disturbed times.

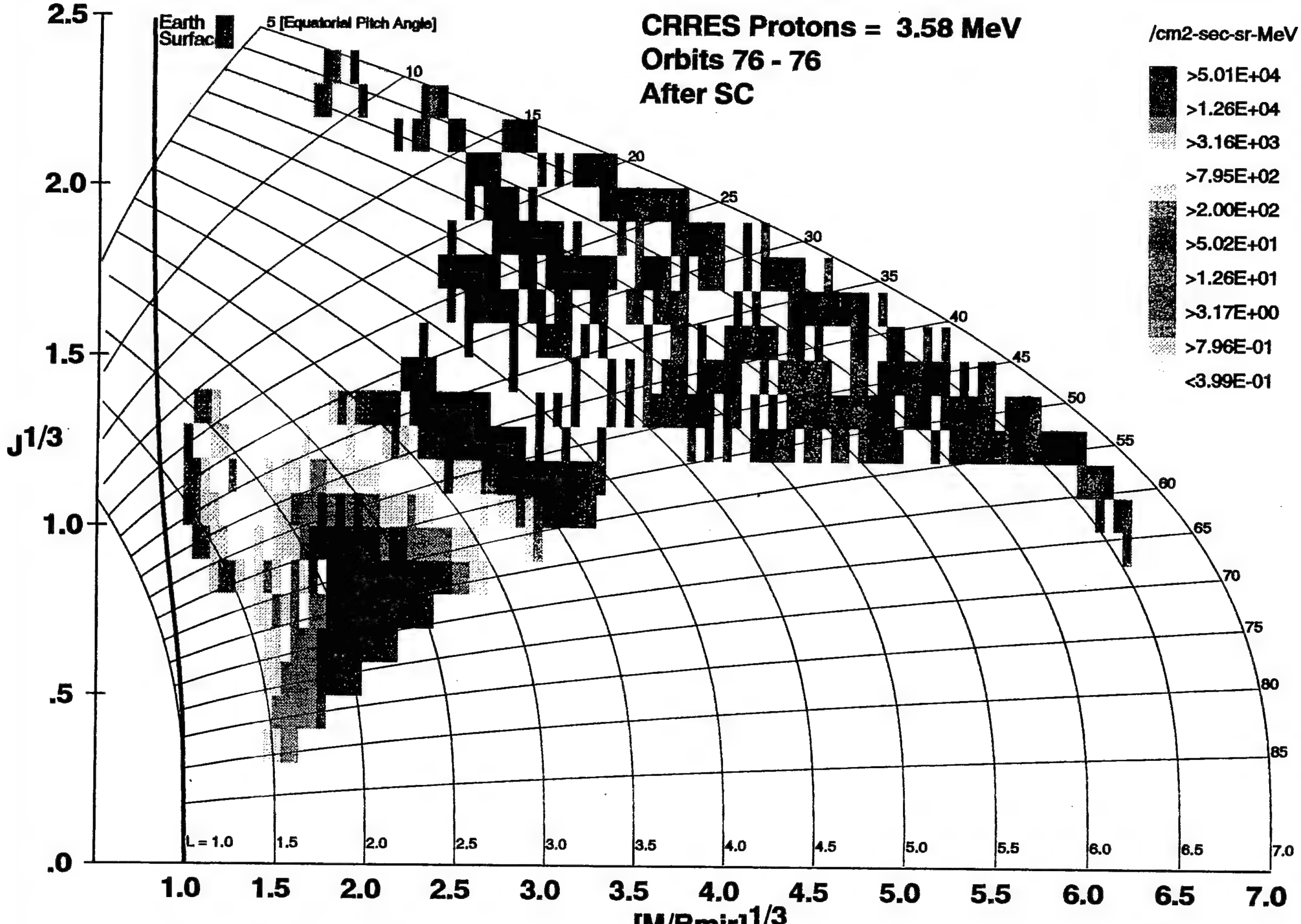


Figure 11. Proton flux distribution after sudden commencement. Cutoff is about 4.5

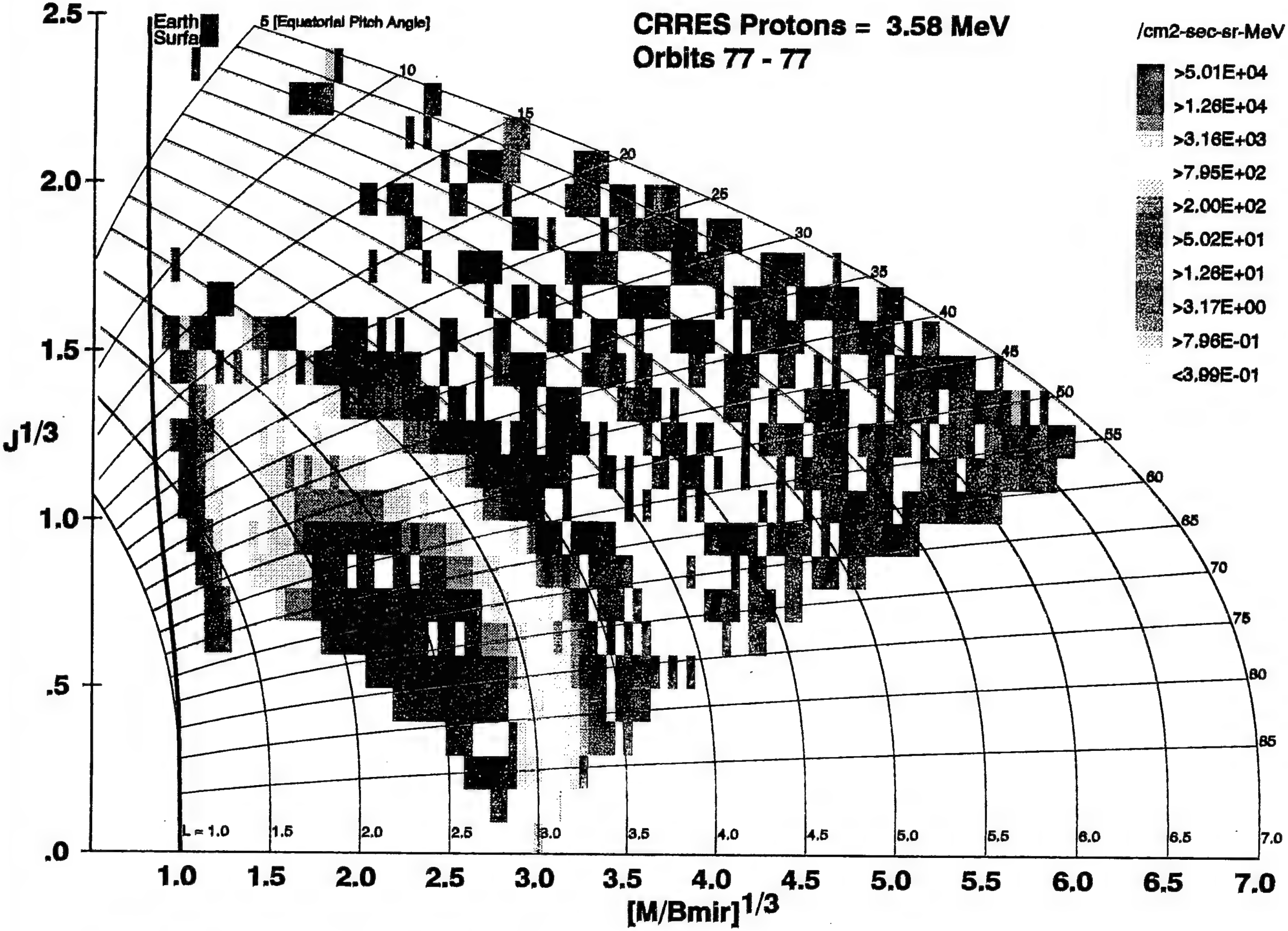


Figure 12. Proton flux on next orbit when standoff distance was still about 7.0 Re.

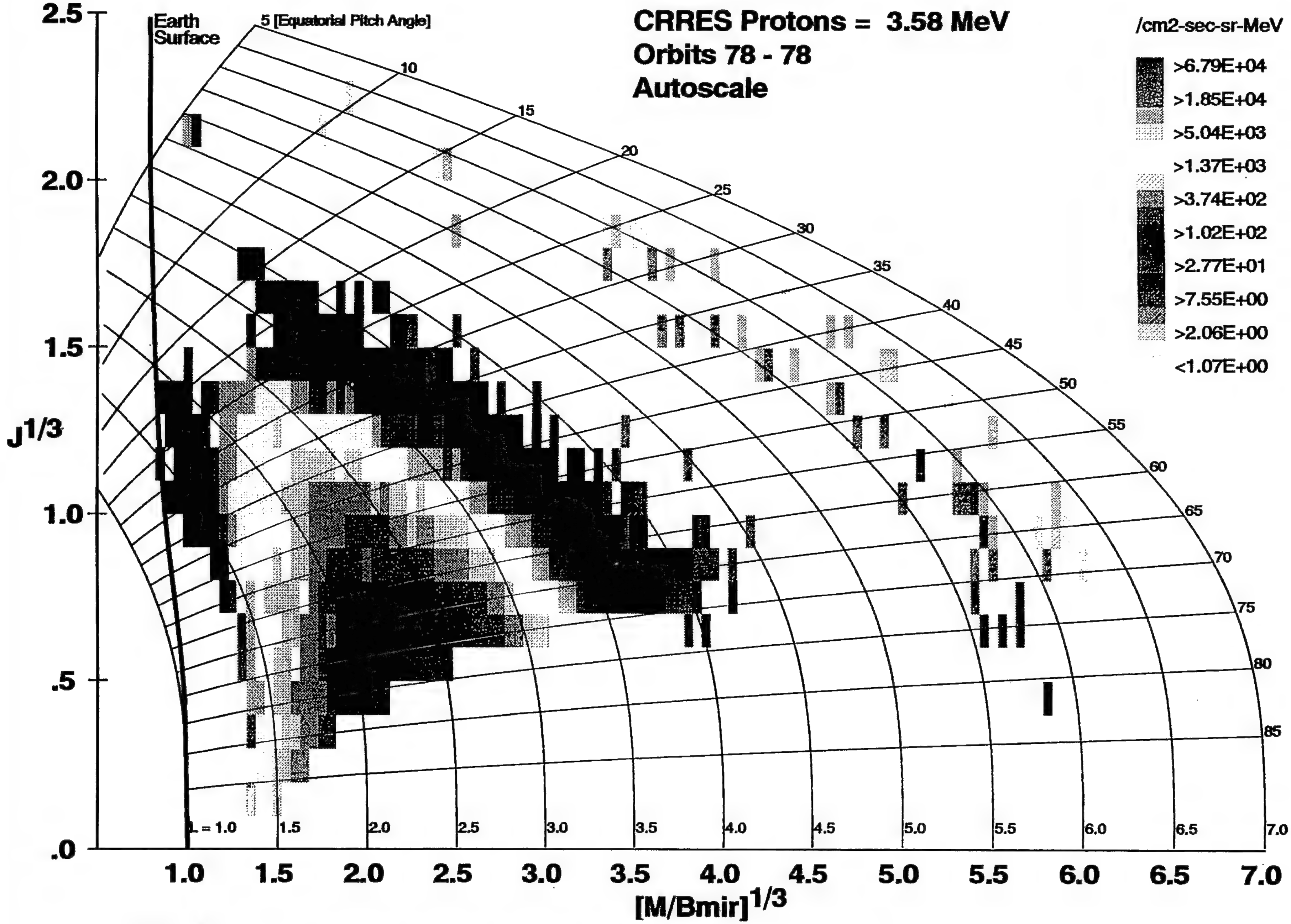


Figure 13. Proton flux after standoff distance has returned to about 10 Re.

3.0 An Electromagnetic Model of the Magnetosphere

In March 1991, a huge sudden commencement changed the entire character of the inner radiation belt. A new radiation belt was created and large numbers of high energy particles were created. In order to study and attempt to understand the acceleration mechanism, a complete time dependent electromagnetic (electric field and magnetic field model) was developed for the CRRES effort. The effort begins using a vector potential representation of the magnetic field developed under and earlier effort.

The vector potential model was developed more than 15 years ago, and is the starting point for the present analysis. One should remember that the current systems that are used for its definition are the same current systems that are used for the highly successful 1977 magnetic field model. The original vector potential model developed in 1977 contained the effects of all of the current systems and thus the functions used in the fit are unnecessarily complicated for this work. For the study of the acceleration due to the changing magnetopause currents, one should use only the magnetopause currents. It was possible to easily separate the coefficients for the magnetopause currents. Redefining the functional form to remove terms that help fit the ring current would have been very labor intensive. Although simpler functions would increase computational efficiency and accuracy, the precision required for this initial study did not justify this additional work. Thus, the vector potential model used for this study consist of a set of polynomials and polynomials times an exponential that has virtually the same form as the 1977 magnetic field model. The coefficients for the model are the coefficients derived from the magnetopause currents. As discussed

above, the accuracy of this vector potential model was validated in 1977 when the curl of the vector potential was compared point for point against the magnetic field values calculated from the current systems and the total calculated magnetic field was compared to the delta B contours of Sugiura (Sugiura, et. al., 1971). The vector potential model listed in Appendix E is thus a high fidelity model of the vector potential developed from the 1977 tilt dependent model and specifically modified for this effort to include only the magnetopause currents.

The vector potential model developed from the 1977 tilt dependent current system is only valid during quiet time. This model was extended for use during disturbed times using techniques developed during the Consolidated Data Analysis Workshops (CDAW). Extensive work with the various (CDAW) data sets validated a method of extending a quiet time model to disturbed times. This method has been shown to work particularly well for scaling the magnetopause currents in response to changes in the stand-off distance. The justifications for the scaling techniques are discussed in Olson and Pfitzer 1982. As part of the CRRES study, we revisited the scaling rules for the magnetic field models and for the vector potential model. We found that the scaling rules used during CDAW were correct for the magnetic field but incorrect for the vector potential.

3.1 Scaling the Vector Potential

The vector potential is given by

$$\vec{A}(P) = \frac{\mu_o}{4\pi} \oint \frac{\vec{\sigma}(x, y, z)}{|r - r_p|}\, dA \qquad (15)$$

using the scaling defined in Section 2.4.3 we can show that

$$\vec{A}(P) = \frac{I_M(S)}{S}\left[\frac{\mu_o}{4\pi}\oint\frac{\vec{\sigma}(x',y',z')}{|r'-r_p'|}\,dA'\right] \tag{16}$$

Thus, the Vector potential at point P in the compressed magnetosphere can be obtained by calculating the vector potential in the quiet magnetosphere at scaled point P' and multiplying by scale factor $(10.5/R_S)^2$.

We note that the scaling for the vector potential is the square of $10.5/R_S$, whereas the scaling of the magnetic field is the cube of $10.5/R_S$. Some of the early work in this effort, as well as some of the CDAW work, used the incorrect scaling factors.

The vector potential routine is listed and described in Appendix E.

3.2 The Induction Electric Field

The great advantage of a vector potential model is that one can calculate the Induction electric field directly. The induction electric field, $\mathbf{E}_I$, is given by

$$\mathbf{E}_I = -\frac{\partial \mathbf{A}}{\partial t}. \tag{17}$$

Note that all work in this analysis is performed in MKS units.

To calculate the time dependent electric field one must have a time dependent vector potential. The time dependent vector potential routine, AXYZDN, described in Appendix F gives the vector potential as a function of the standoff distance. Thus, a time-dependent standoff distance will produce a time dependent vector potential. This vector potential can then be used to calculate a time dependent induction electric field.

3.3 Pressure Balance During the March 1991 Event

To develop a model for the magnetic field and for the induction electric field during the large sudden commencement of March 1991, we need a time history of the solar wind pressure during the event. Unfortunately, very little solar wind information is available for this event. We thus assume that before the sudden commencement, the solar wind pressure was nominal and that the standoff distance was at 10.5 Re. We also assume that there was a discontinuous change in the solar wind pressure, a step function in the solar wind pressure. Travel times of the solar wind plasma from sun suggest that the velocity of the shock during the March 1991 event was on the order of 1450 km/sec. We further assumed that the magnetopause was compressed to a minimum distance of about 5 Re. The magnetopause cannot instantly respond to a pressure discontinuity and thus some time is required for the standoff distance to change from 10.5 to 5 Re. The time dependent standoff distance function was first developed such that the calculated magnetic signature matched the signature observed by the CRRES magnetometer. The CRRES magnetometer sees a positive dB/dt lasting for approximately 30 seconds (see Figure 14). Thus, the initial work assumed that the standoff distance moved from 10.5 Re to 5 Re in 30 seconds. The time rate of change of the standoff distance change was assumed to be nonlinear. Since the magnetic field becomes stronger as the field is compressed, the rate of change of distance should slow down as the boundary moves in. Several functional forms were constructed. The final function was selected on the basis of correctly modeling the dB/dt observed by the CRRES magnetometer. The function that was found to give an acceptable result has the form

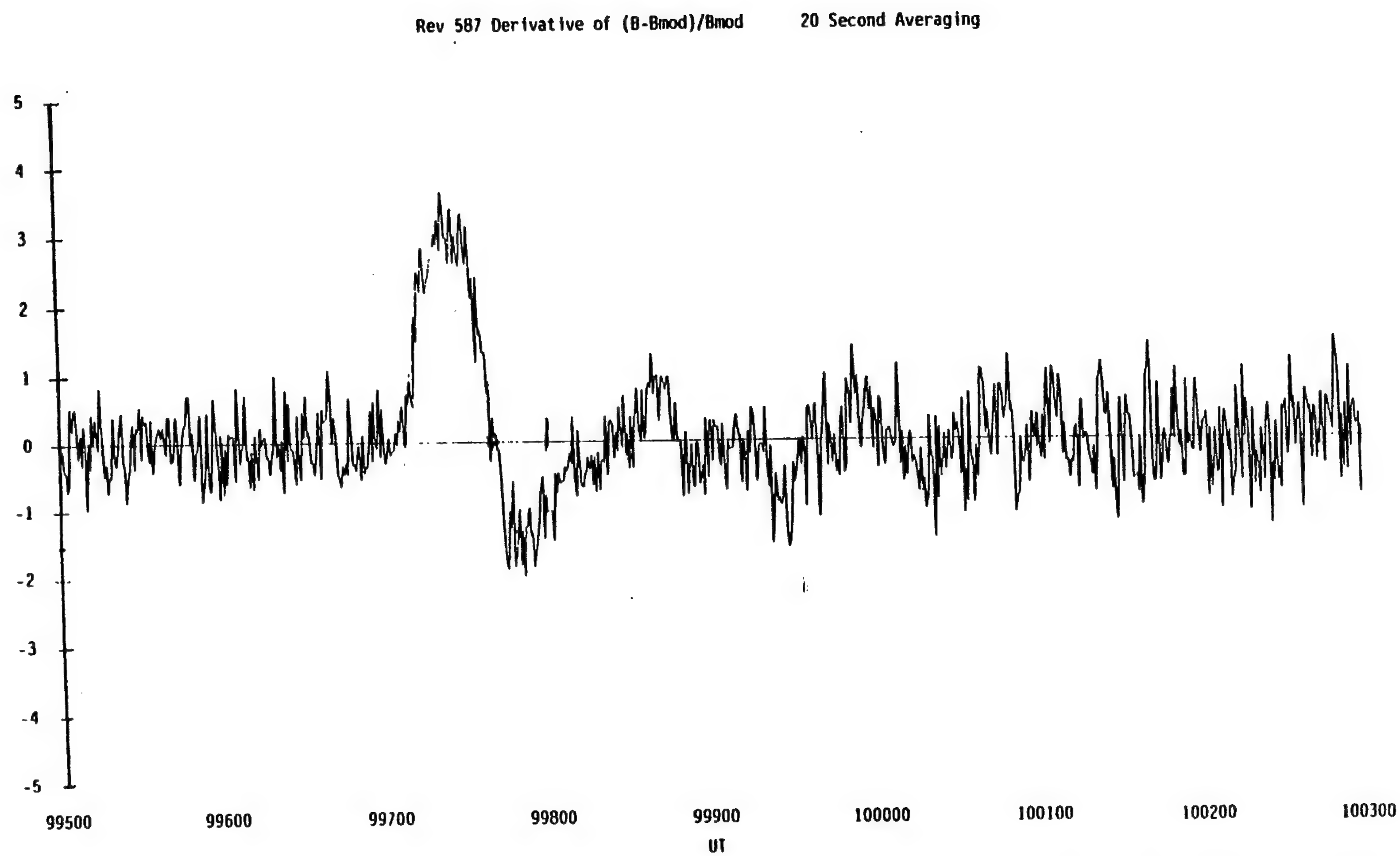


Figure 14. CRRES magnetometer data. Partial derivative with respect to time. Not: Field rises rapidly with time, stays up for 30 seconds, then rate of change reverses direction, but not completely.

$$R_S = 10.5 \cdot \left[1 + 9 \cdot \left(\frac{t}{30}\right)^{1.3}\right]^{-1/3}, \tag{18}$$

where R_S is the standoff distance and t is the time since the start of the event. Figure 15 gives the time dependent standoff distance predicted by the above equation. Using this standoff distance function for the vector potential model gives a dB/dt at the location of CRRES as shown in Figure 16. This compares very favorably with the CRRES data shown in Figure 14.

The above determined of rate-of-change of standoff distance has no theoretical foundation. George Siscoe of UCLA (private communication) noted that during a sudden change in solar wind pressure, the magnetosphere must remain in equilibrium at all times. That is, when the solar wind pressure changes, the magnetospheric boundary must move so as to maintain pressure balance during the motion. The velocity term in the pressure balance equation is not the velocity of the solar wind, but the difference in the velocity of the solar wind and the velocity of the moving boundary. Thus,

$$R_s = 98 \cdot \left[\rho V^2\right]^{-1/6} \tag{19}$$

where v is the velocity difference between the solar wind speed and the velocity of the boundary. The constant 98, is the constant that was developed for our dynamic magnetic field models. This can be rewritten to give

$$V = \frac{1}{\sqrt{\rho}}\left[\frac{98}{R_S}\right]^3 \tag{20}$$

Thus,

$$v_S + \gamma\frac{\partial r}{\partial t} = \frac{1}{\sqrt{\rho}}\left[\frac{98}{r}\right]^3 \tag{21}$$

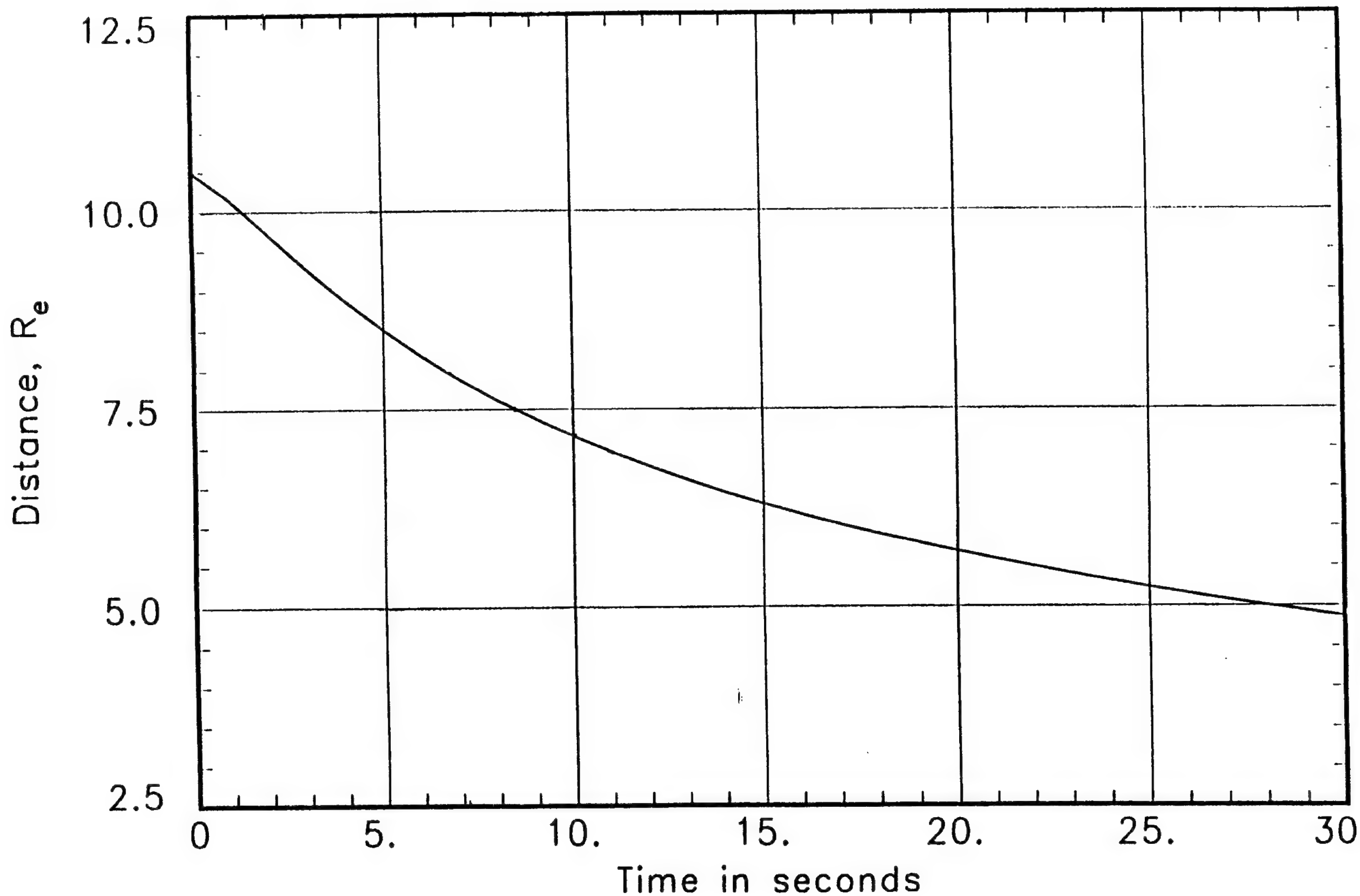


Figure 15. Standoff distance as a function of time. Rate of change of distance was adjusted to give good fit to the rate of change of the magnetic field as observed by the CRRES magnetometer.

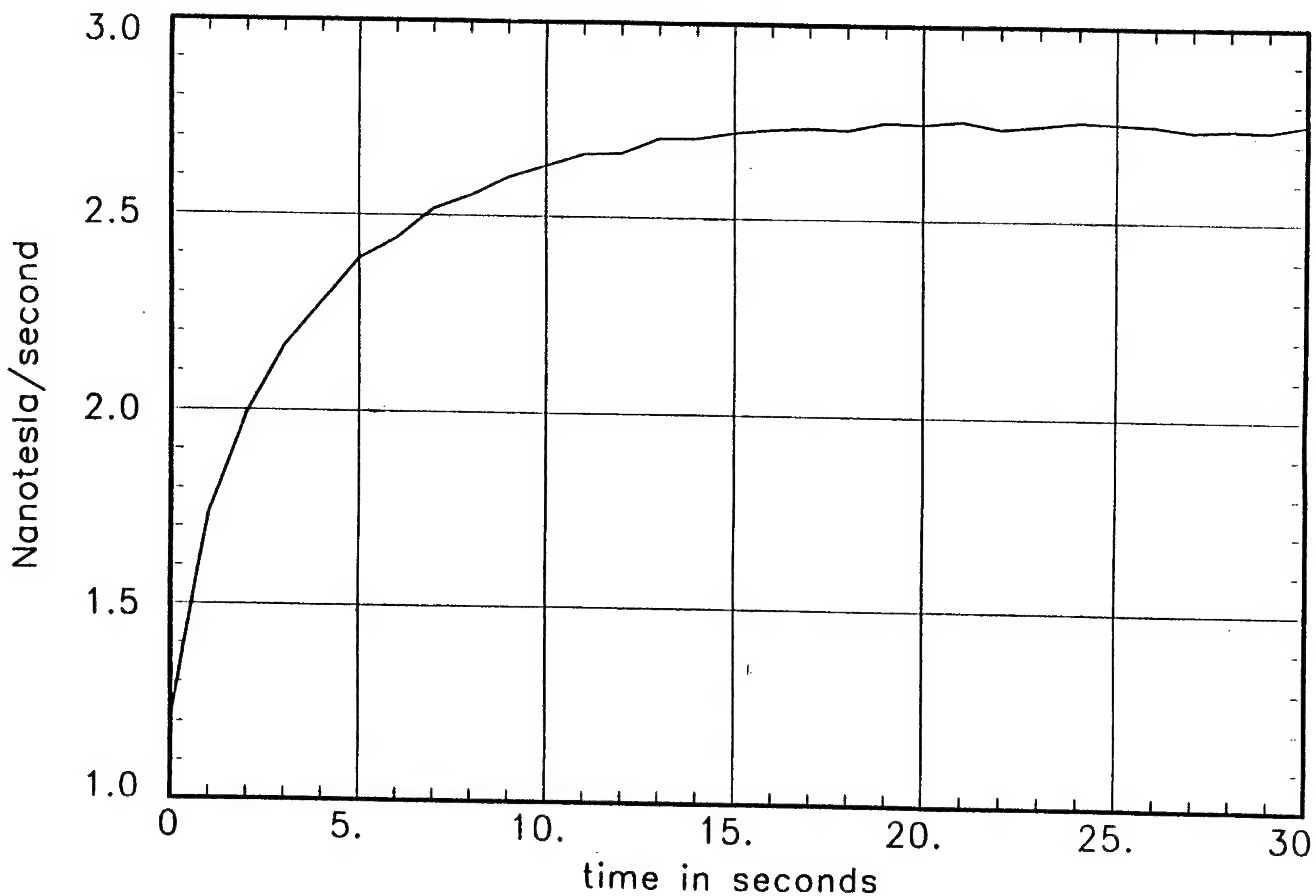


Figure 16. Time rate of change of the magnetospheric magnetic field at the location of CRRES. This field change is calculated using the standoff distance shown in Figure 15.

where v_S is the solar wind velocity, $\partial r / \partial t$ is the velocity of the boundary, γ is 6371 and changes the boundary velocity from units or R_e/sec to km/sec, r is the instantaneous position of the boundary. This gives rise to the following simple differential equation

$$dt = \frac{\gamma r^3 \, dr}{\frac{98^3}{\sqrt{\rho}} - v_s r^3} \tag{22}$$

This can be written as an easy to solve integral

$$t = \gamma \int_{10.5}^{R_s} \frac{r^3 dr}{a + br^3} \tag{23}$$

where $a = 98^3 / \sqrt{\rho}$ and $b = - v$

Evaluating the integral gives

$$t = \frac{r}{a}\Big|_{r=10.5}^{r=R} - \frac{k}{3b}\left[\frac{1}{2}\ln\frac{(k+r)^2}{k^2 - kr + r^2} + \sqrt{3}\tan^{-1}\frac{2r-k}{\sqrt{3}k}\right]\Bigg|_{r=10.5}^{r=R_s} \tag{24}$$

where k^3=a/b

This gives the time, t, since the start of the arrival of the solar wind pressure change as a function of the instantaneous location of the standoff distance R_S. Since no value was available for ρ the solar wind number density, we used a value of 30 which is consistent with a minimum standoff distance of 5 at a solar wind velocity of 1450 km/sec. Substituting values into the above equation gives the time dependent standoff distance as a function of time determined using dynamic pressure balance. The result of this dynamic pressure balance analysis is given in Figure 17. We note with interest that this figure is very similar to Figure 15. Figure 15 is determined by attempting to fit the CRRES

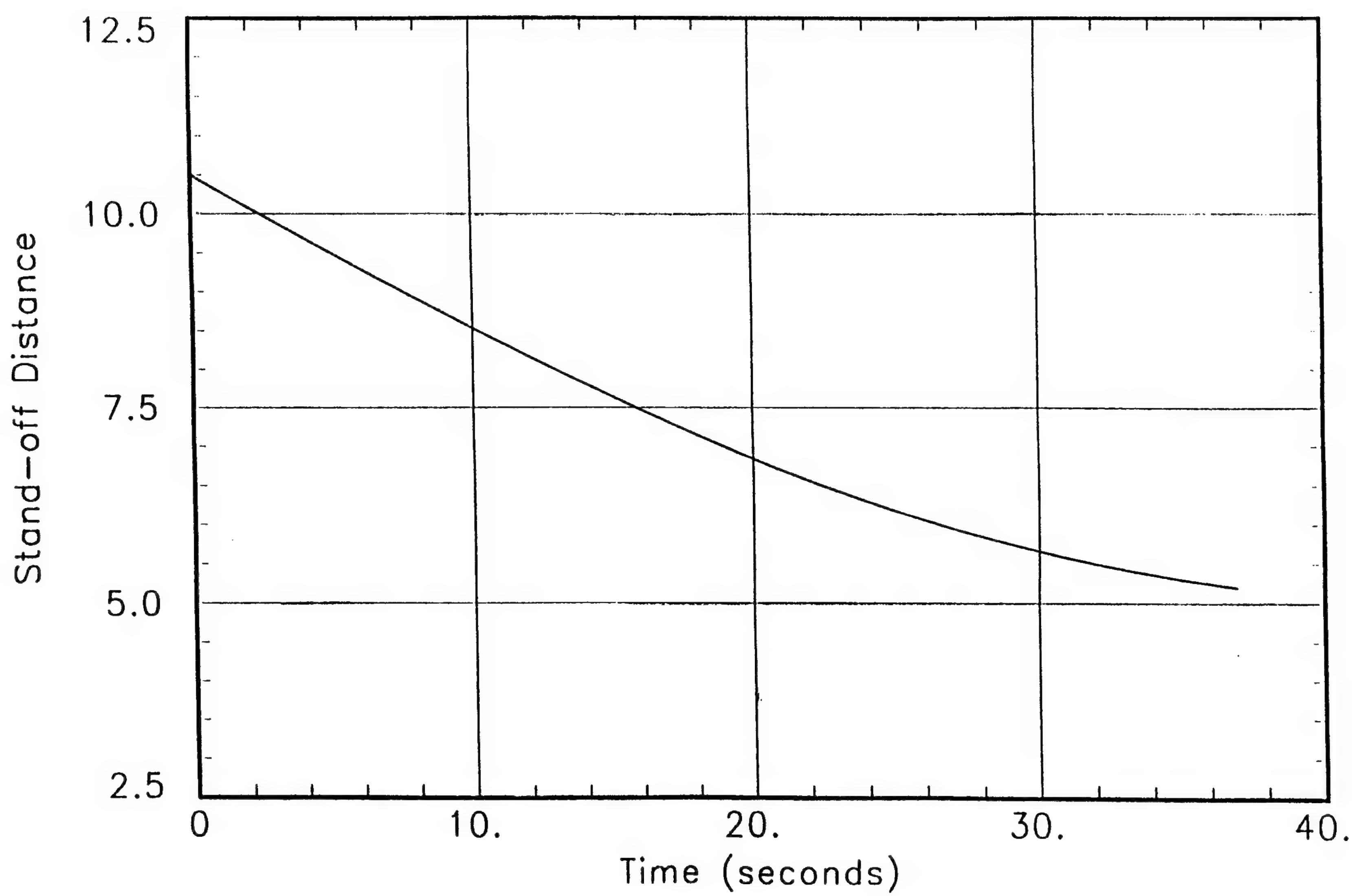


Figure 17. Time rate of change of standoff distance calculated using dynamic pressure equilibrium. Pressure balance is maintained during boundary position change.

magnetometer observations and Figure 17 attempts to use a more theoretical approach. Both methods could be substantially improved if actual measurements become available of the real solar wind velocity and particle density during this event. This analysis is, however, consistent enough to allow us some confidence that either Figure 15 or 17 can be used as the starting point for the development of a time-dependent vector potential. For this report we have used the much simpler form (equation 18). The time dependent vector potential is driven by the time dependent standoff distance. This allows us to investigate the induction electric field created by the change in location and the change in strength of the Chapman Ferraro currents. During this compression, the Chapman Ferraro currents move inward from 10.5 to 5.5 R_e and increase in strength by an order of magnitude in a time period of approximately 30 seconds.

3.4 Magnetic signature During the March Event

Figure 16 gives the derivative of the magnetic signature determined by the time dependent magnetopause model. The integral of this magnetic field change, the amplitude of actual delta **B** spike predicted by the model at the location of CRRES, is 75 nanotesla. At 2.7 R_e on the noon meridian, the delta **B** spike is predicted to be 240 nanotesla. At the surface of the earth, the magnetic field spike should vary from 120 nanotesla at midnight to 170 nanotesla at local noon. Mid-latitude magnetometers may see a magnetic field spike that will exceed this magnitude due to currents induced in the earth. The induction currents due to the conducting earth may increase the magnetic field signature as much as 60 percent. The exact enhancements have not yet been calculated. We understand the size of the increases to the S_q signatures that have a period of 24 hours. It is not prudent, however, to apply the same increase to a change with the period of 30 seconds. In order to determine the induction currents in the

surface of the earth, one must solve the problem of a magnetized conducting sphere with finite conductivity during a 30 second magnetic field pulse. This is a non-trivial problem that may require extensive analysis.

3.5 The Induction Electric Field During the March Event

The time dependent standoff distance given in Figure 15 was used to calculate the induction electric field during this event at various locations within the magnetosphere. Figure 18 gives the induction electric field at CRRES during the 30 second compression period. One notes the rapid rise in the electric field to a level of approximately 50 mV/m. The present model only represents the period of active inward motion. At the end of the compression period the electric field will rapidly decay to zero and then actually reverse since the magnetosphere relaxed somewhat after the initial compression (see Figure 14). Figure 19 gives the induction electric field on the local noon meridian at a distance of 2.7 Re from the center of the Earth. One notes that this field increases to almost 400 mV/m. The rate of change of the induction electric field in Figure 19 differs from the rate of change seen in Figure 18. This is due to the fact that Figure 19 is on the noon meridian and much closer to the approaching currents. The electric field at this location is due not only to the increasing strength of the Chapman-Ferraro currents but also to the rapidly approaching currents. CRRES which is toward the dark side of the earth is farther from the currents and thus is most sensitive to changes in the strength of the Chapman-Ferraro currents. Figure 20 is a snapshot of the electric field in the equatorial plane at time t = 20 seconds. At this time the magnetopause boundary is passing through 6.0 R_e. The induction electric field is given every 1.0 R_e on a rectangular grid. The length of the line is proportional to the strength of the field. A line .5 R_e long corresponds to a field value of 500 mV/m. The direction of the

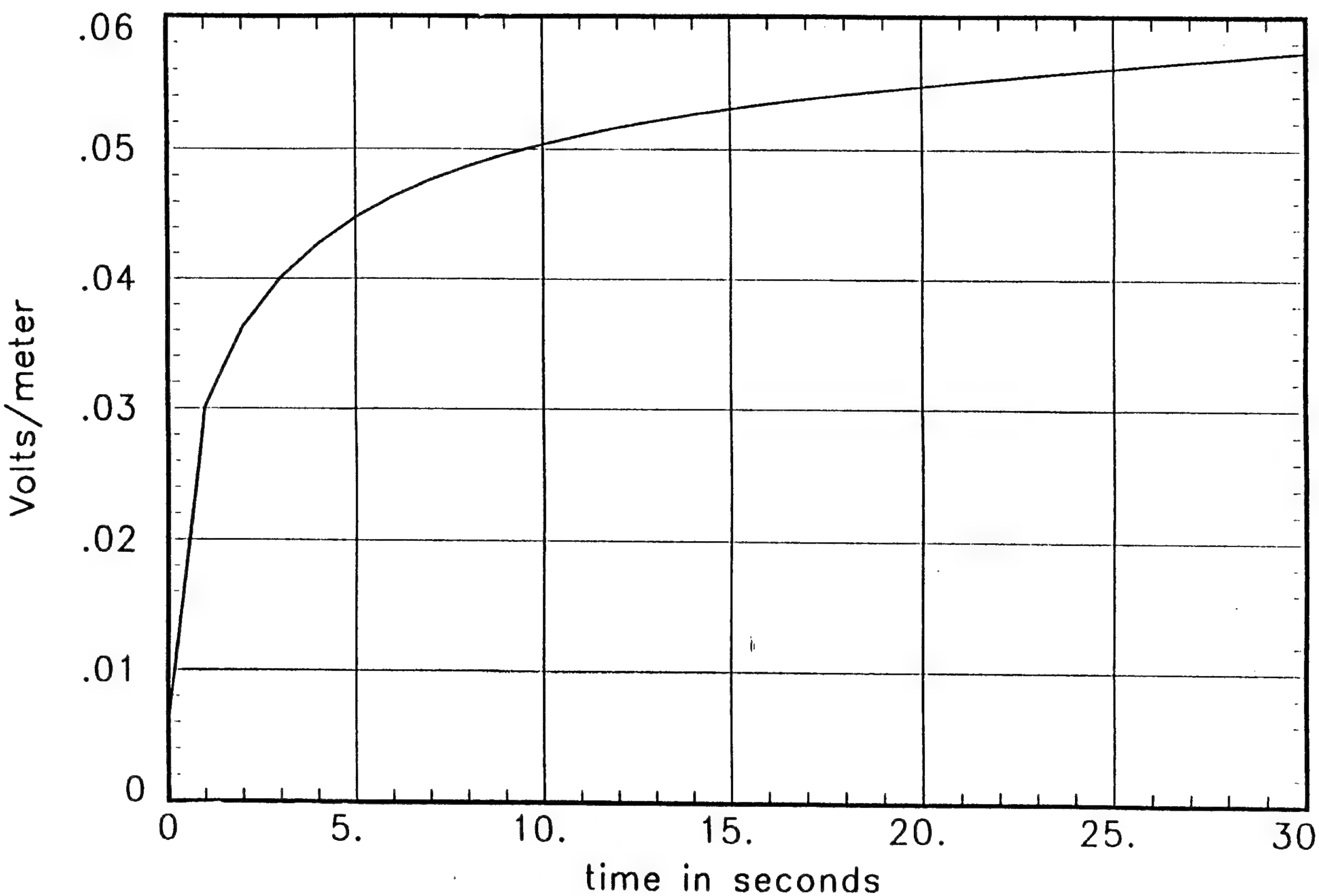


Figure 18. Induction electric field at the location of CRRES. Since CRRES is far from the boundary, the primary contribution to this change is the change in the strength of the Chapman-Ferraro currents.

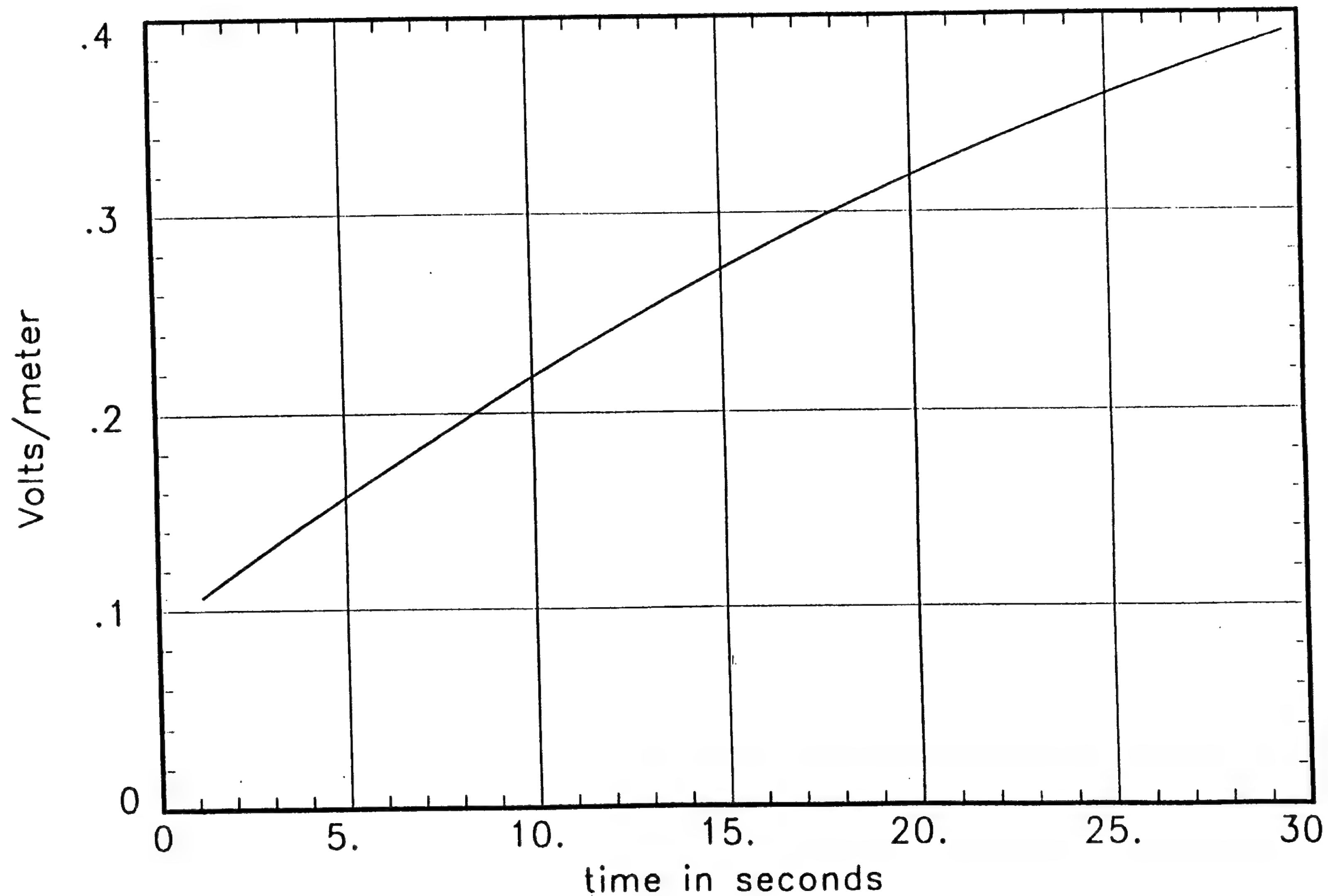


Figure 19. Induction electric field on the sun earth line at a distance of 2.7 Re from the earth. Change is due to the increase in the Chapman-Ferraro currents and to the reduced distance from the observation point to the currents.

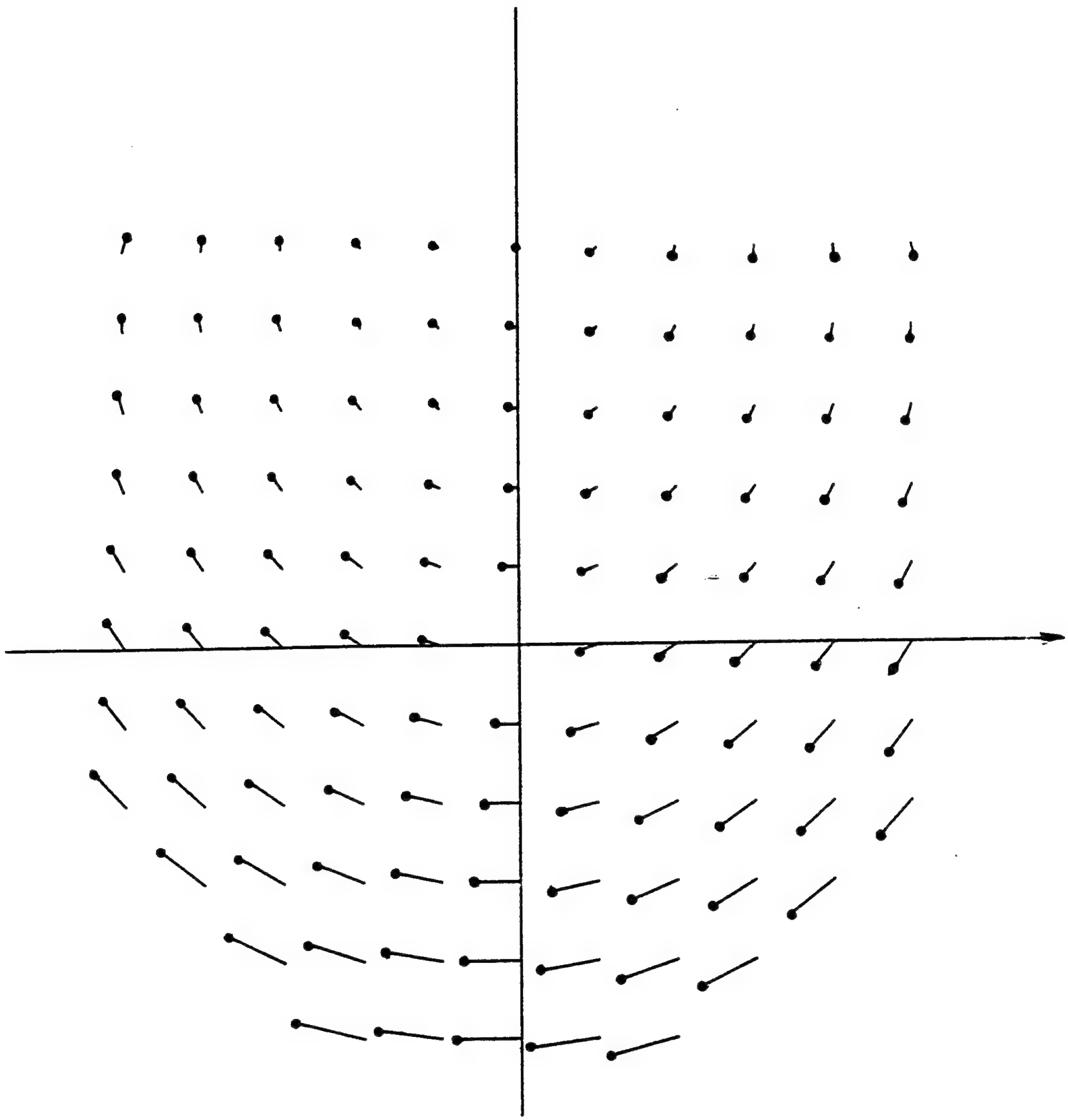


Figure 20. Electric field in the magnetic equatorial plane. Time is t = 20 seconds. Standoff distance is at 6 Re. Electric field is evaluated on a 1.0 Re grid. A vector 1.0 R3 long corresponds to a field of 0.5 V/m. Dot at the head of the vector points in the direction of the field. +X is down and toward the sun.

line gives the direction of the induction electric field vector. The dot at the end of the line points in the positive field direction. We note that the field is in most places tangent to the azimuthal direction and that the direction of the field on the sunlit side is such that both electrons and protons will experience a large gain in energy. There will be deceleration near local midnight, but this is small since the field is much smaller in this region. Once the compression of the magnetic field stops and the magnetosphere relaxes the electric field pattern will reverse its direction. From the magnetometer data in Figure 14, one can see that only a partial relaxation occurs and thus the deceleration fields will be weaker. Furthermore, any particle that was near noon during the start of the acceleration will most likely be near local midnight during the deceleration phase and will thus be shielded from the deceleration phase. The induction electric field is a non-conservative field. Even if the acceleration and deceleration fields were equal and opposite, some of the particles would experience substantial permanent acceleration. There would of course be a class of particles that would experience permanent deceleration.

3.6 The Parallel Electric Field

Because of symmetry, the induction electric field from the magnetopause currents is perpendicular to the magnetic lines of force in the magnetic equatorial plane. At all other locations there is a component of the induction electric field that is parallel to the field lines. Since the conductivity along field lines is very high, there will be a very rapid redistribution of charges along the line of force such that the total electric field parallel to the lines of force is zero. The total electric field is given by

$$\mathbf{E}_{\mathrm{T}} = -\frac{\partial \mathbf{A}}{\partial t} - \overline{\nabla}\Phi \qquad (25)$$

$-\overline{\nabla}\Phi$ is the scalar potential electric field is due to charge separation. Charges will realign themselves such that the parallel component of $\mathbf{E}_T$ is everywhere zero. This charge rearrangement which cancels the parallel portion of the total field can substantially modify the electric field perpendicular to the lines of force.

It is possible to calculate the charge separation electric field. During our work with the induction electric field due to wobbling dipole we developed a routine that performed a line integral along a line of force into the ionosphere. The total electric field along this line of force was required to remain zero everywhere along the line of force. This required introducing a potential variation along the line of force so that the gradient of the potential along the line of force would everywhere cancel the parallel component of the induction electric field. Adjacent line integrals can then give the gradients of the potential electric field perpendicular to the lines of force. Since potentials are arbitrary to within a constant, the analysis depends on the correct use of the boundary condition. Since the foot of the field line is anchored in the conducting ionosphere, the ionosphere becomes the physical boundary condition. During our wobbling dipole analysis, we used either an equipotential ionosphere or an ionospheric boundary condition that assumed that the earth was a rotating conducting magnetized sphere.

A similar analysis will be instructive in this case. This part of the electric field analysis has not yet been completed. It is the intention to place a high priority on this analysis. During our wobbling dipole analysis, the induction electric field was very small and the difference between the two boundary conditions was very large. Since the driver induction electric field for this event is two orders of magnitude greater than that of the wobbling dipole field, we expect less sensitivity to the form of the ionospheric boundary condition. We do, however,

expect a substantial change in the overall electric field pattern. In many cases, canceling the parallel electric field may substantially increase the perpendicular electric field.

4.0 Particle Acceleration

Since the electric field is approximately azimuthal in the equatorial plane one can make a quick estimate of the amount of energy gain that one can expect for protons and electrons. The energy gain is simply the $\mathbf{E} \cdot \mathbf{dl}$. Estimating the path length from local dawn to dusk or dusk to dawn and multiplying by approximately 400 mV/m gives an approximate energy gain of 20 MeV at R =2.7 and 30 MeV at R = 4. These are very large numbers and suggest that the induction electric field due to the Chapman Ferraro currents is very important in understanding the particle energization that CRRES observed during the March 1991 event.

4.1 Lorentz Force

The force on a charged particle is given by

$$\mathbf{F} = q\ (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{26}$$

A modified cosmic ray trajectory code was used to integrate the trajectory of protons using equation 26. The initial cosmic ray code used by many investigators was modified to step in time instead of position. It was modified to include the effects of the electric field and energy conservation was removed from the code. The code can perform on the order of 50,000 integration steps before round off errors begin to affect the accuracy of the code. Thus, proton motion during the event can easily be studied. However, the motion of electrons cannot easily be studied by a trajectory integration program. To study electron motion a guiding center code must be used. For this analysis only proton trajectories were studied.

4.2 Particle Motion During the March Event

A Lorentz force trajectory code is very attractive because of its simplicity. Cosmic ray codes have been extensively verified and shown to be accurate. The Lorentz force equation includes all effects. For our analysis we used the electric field as given by the induction electric field due to the changing Chapman-Ferraro currents. The electric field is calculated from the time dependent vector potential. Similarly, the magnetic field consists of a dipole field plus the time dependent magnetic field calculated from the curl of the same time dependent vector potential. The magnetic and electric field codes are described in Appendix E. An overview of the Lorentz force integration code is given in Appendix F. The summary of the Lorentz force code is presented in Appendix F for reasons of completeness. The listing of the code will allow the user to easily verify the results of the analysis presented in this document.

Depending on the analysis a particle trajectory was either integrated in the forward direction or the trajectory code was reversed and a negative proton was integrated backward in time. This allows us study the acceleration of particle during the March 1991 event. It was the hope of this analysis to unambiguously show how the new inner radiation belt was created. Are the particles accelerated from the local population or are they accelerated inward from the cosmic ray flux in the outer zone? Both methods were completely investigated

Figure 21 shows a sample trajectory calculation. The particle was started at approximately 3 R_e and 3 hours local time with an energy of 50 MeV and a pitch angle of 90 degrees. The particle was started at time t = 30 seconds. At time t = 0 seconds the proton was at a local time of 21 hours and had an energy of 10 MeV and was at a radial distance of 4.5 R_e. Thus, during the 30 seconds of

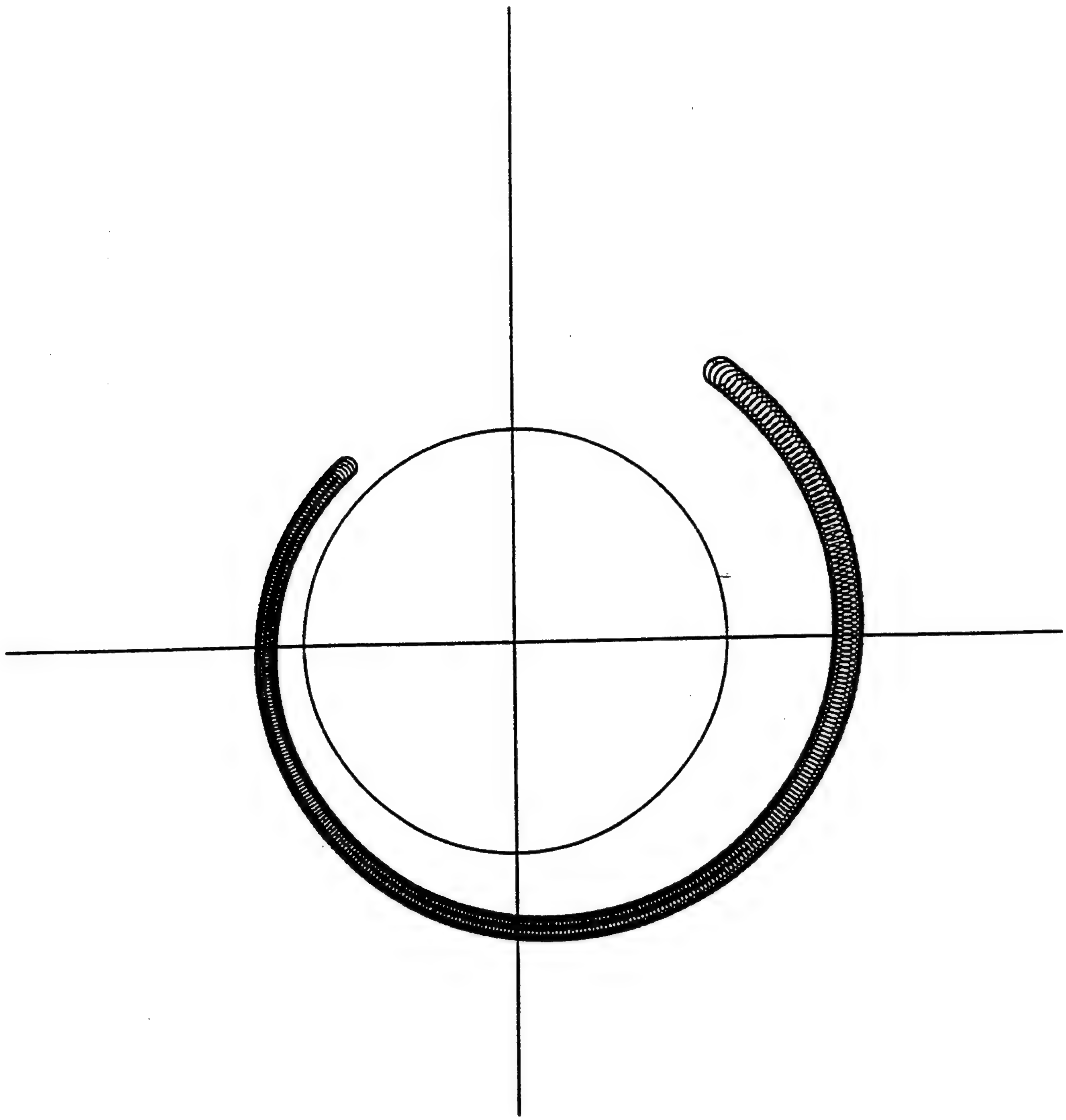


Figure 21. Sample Proton trajectory. This 50 MeV proton is started at 3 hr local time at 3.0 Re. The proton trajectory is integrated backward for 30 seconds. The proton originates at a local time of about 21 hrs with an energy of 10 MeV. The proton gains an energy of 40 MeV. +X is down and toward the sun.

positive dB/dt, this test particle drifted through 270 degrees. From Figure 20 one can see that almost the entire drift period was in a region where the electric field was in a direction necessary for acceleration. Many other trajectories were analyzed. The amount of energy gain is strongly associated with the drift velocity. The particle whose drift velocity is such that it drifts at least 180 degrees in 30 seconds shows the largest amount of energy gain. Very low energy particles have very low drift velocities and thus their $\mathbf{E}\cdot\mathbf{dl}$ is very small since the drift path length in 30 seconds is short. These particles will also be decelerated by the relaxation of the boundary after the initial increase. The particles with fast drift velocity will have drifted to the night side of the earth where the deceleration effect is small, but the slow particles will still be on the dayside and will experience the full deceleration.

Table 6 is a summary of results. The table lists starting and ending Ls as well as starting and ending energies. From this table one can see that energy gain is larger at higher energies and that the energy gain is larger at larger distances. Furthermore, the change in L is less for the smaller L shells. A quick investigation of this table shows that protons with a final L shell of 2.4 originated from Ls in the vicinity of 3.0 Re. These protons most likely originate from the existing trapped proton flux. Protons with a final L of 3.0 originate from Ls greater than 4.5 and could thus have their origin in the solar proton flux present in the outer zone at this time.

The results in Table 6 are all for particles with a pitch angle of 90 degrees. Several trajectories were run for non 90 degree particles. It is apparent from these runs that the energy gain is in the component of energy perpendicular to the magnetic field. Thus, this acceleration mechanism will produce particles with

Table 6

Summary of Accelerations

Final L	Final Energy	Starting L	Starting Energy	Energy Change
2.4	50	2.9	30	20
2.4	30	2.9	15	15
2.4	10	3.0	4	6
2.7	50	3.5	20	30
2.7	30	3.7	9	21
2.7	10	3.9	2	8
3.0	50	4.5	10	40
3.0	30	4.8	6	24
3.0	10	5.0	1	9

a pitch angle spectrum peaked near 90 degrees and thus the newly created belt will be much stronger near the equator.

4.3 Acceleration of Existing Inner Zone Protons

The study of the creation of the new proton belt became an obsession. In the previous section we showed that the energy released during this event was enormous. There was more than enough energy gain in the system to explain the new particles. Thousand of trajectories were run and the original proton environment was mapped to its new configuration. The results were disappointing.

Protons were moved from more populous lower energies to higher energies just as expected. The protons moved inward to lower L's. This, of course, maps the less populous higher L shells to the more populous lower L shells. Furthermore, only a small fraction of the protons were in the correct phase to fully participate in the acceleration process. Even with the large number of trajectories run the percent efficiency of the acceleration is at best a guess. The best estimate of the number of particles that were actually accelerated is on the order of 20 to 30 percent. There were also some number of protons that were in an unfavorable phase relationship and actually lost energy during this event. When the results of the various trajectory runs were summarized and applied to the pre-event inner zone, small increases in the number of protons were seen at all energies. There was no hint of the very large new inner peak. Figure 22 summarizes the result of this extensive investigation.

Although, the induction electric field program cannot be used to study the trajectories of the inner zone electrons, back of the envelope calculations seem

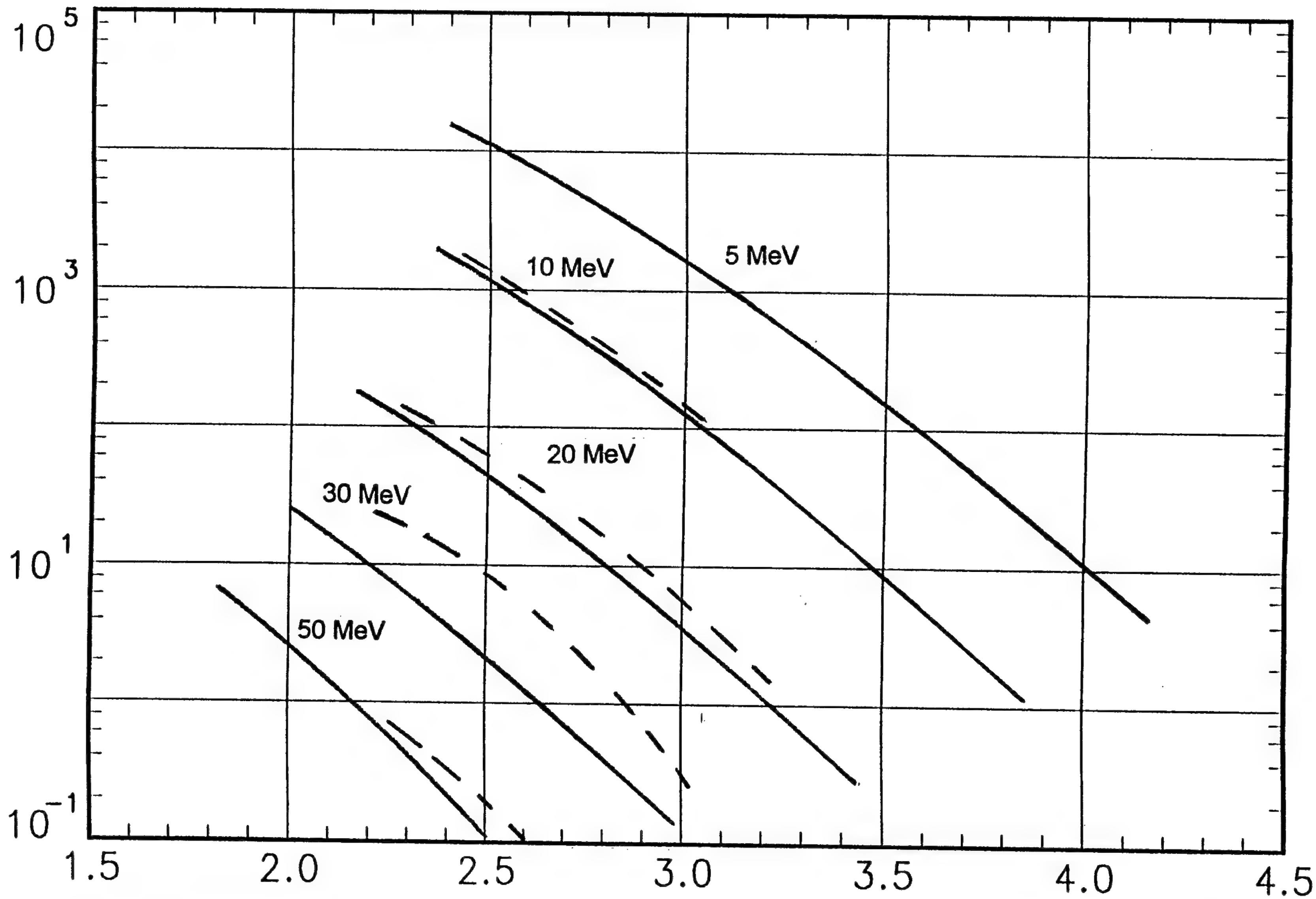


Figure 22. Estimates of maximum increase in proton flux by March 1991 event. Solid lines are the L dependent curves that were input to the energy gain program. Dash lines are approximate idealized increases due to the acceleration of the induction electric field created by the March Sudden commencement.

to indicate that the induction electric fields effect on existing inner zone electrons may explain the rearrangement seen in the inner zone electron flux.

4.4 Acceleration and Entry of Cosmic Ray Protons.

As seen in Figures 7 and 8, there is a ready supply of cosmic ray protons deep inside the magnetosphere, at Ls as low as 5.5 to 6.0 when the magnetosphere is in its relaxed configuration. When we begin our integration with this configuration and start with protons on the L=5.5 drift shell, we can move protons down to L=3.5 with intensities sufficient to explain some of the observations. It is possible to get a peak since protons that were initially at the right energy at the right location are not completely decelerated when the field relaxes. There is a maximum penetration which does form an inner edge; but once again, were having problems coming up with a cohesive picture. The location of the peak, the restricted energies that worked are not completely consistent with the observations. The encouraging part of the picture was that we did get a peak with an inner edge, although it was at a much larger L an L of about 3.5.

There is a third mechanism that is more likely also important. As the magnetosphere is compressed, the direct entry of particles to the lower L drift shells has already been shown in Figures 11 and 12. Protons enter on the daylit dawn side of the magnetosphere and after entry drift through the midnight region. Protons entering during the time of maximum compression when the standoff distance is 5 Re can be show by trajectory calculations to get to Ls as low as 3.3. The protons that are on the night side of the magnetosphere when the field relaxes see limited deceleration and become permanently trapped when the field relaxes. Once again, it is possible to get a peak. This method of entry

which does not so strongly depend on the effects of the induction electric field is able to more easily explain the spectrum of the new peak. The induction method is very strongly biased to those energies having the correct drift velocity. The direct entry and subsequent trapping by the non conservative changes affects all protons. The highest energies will penetrate the deepest thus the higher the energy the lower the L value of the peak. This method needs extensive additional trajectory runs. We were not able to fully investigate these effects in the time that was available to the project. We are still on a time available basis, attempting to extend this work to obtain quantitative results.

This study has been one of extreme frustration. There is so much energy available, but the actual data is sparse. Figures 23 and 24 show the Dst and the standoff data that are available for this event. The key data, the standoff distance is missing. There are suggestions that the magnetopause boundary may have dropped to as low as 4.0. If this is indeed the case then the direct entry without extensive induction accelerations may end up to be the most probable source. Induction effects are still somewhat important in determining the final energy spectrum and the final L distribution of the protons, but they may not be the main driver for the actual creation of the new peak.

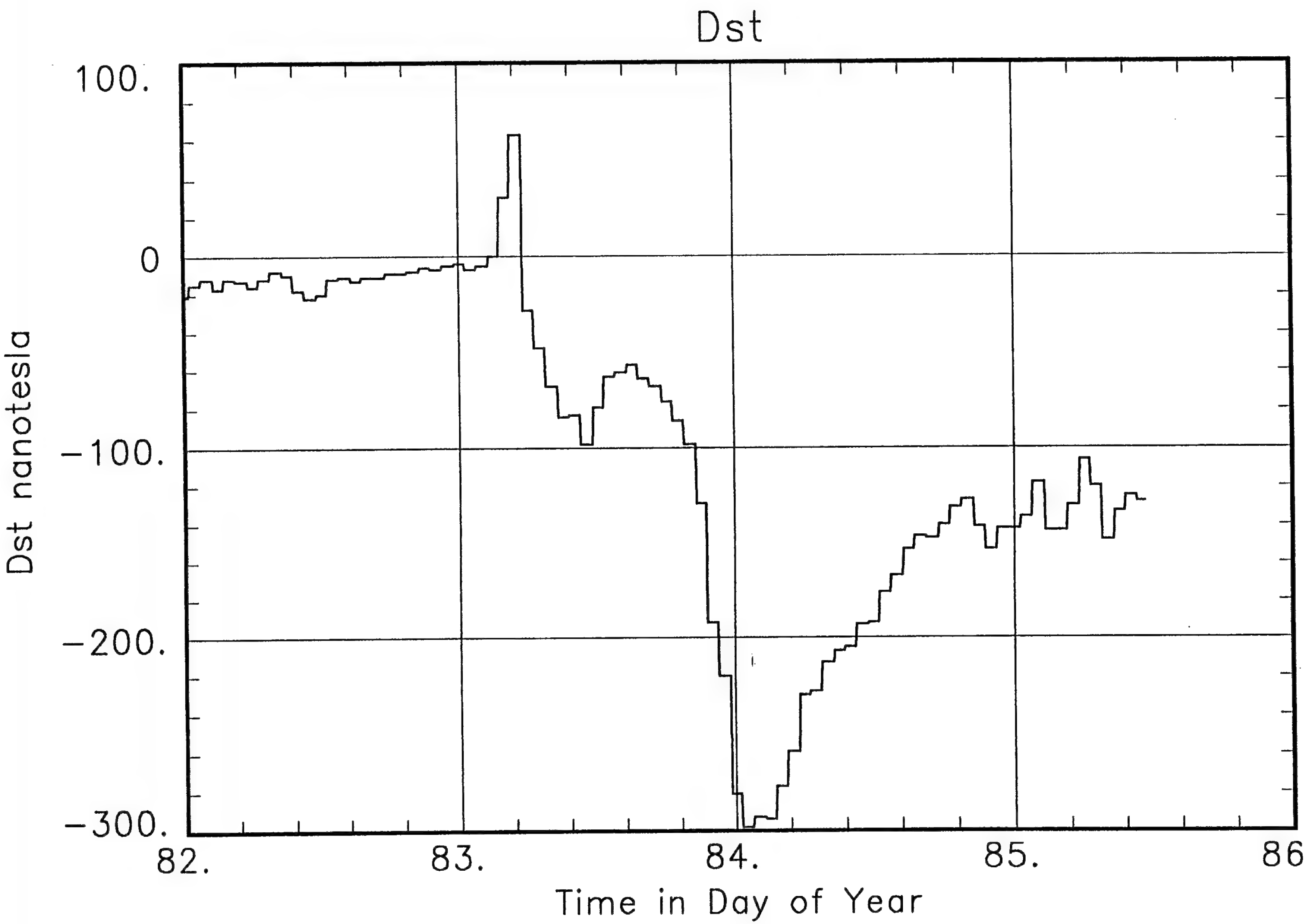


Figure 23. Plot of Dst during March 1991 event.

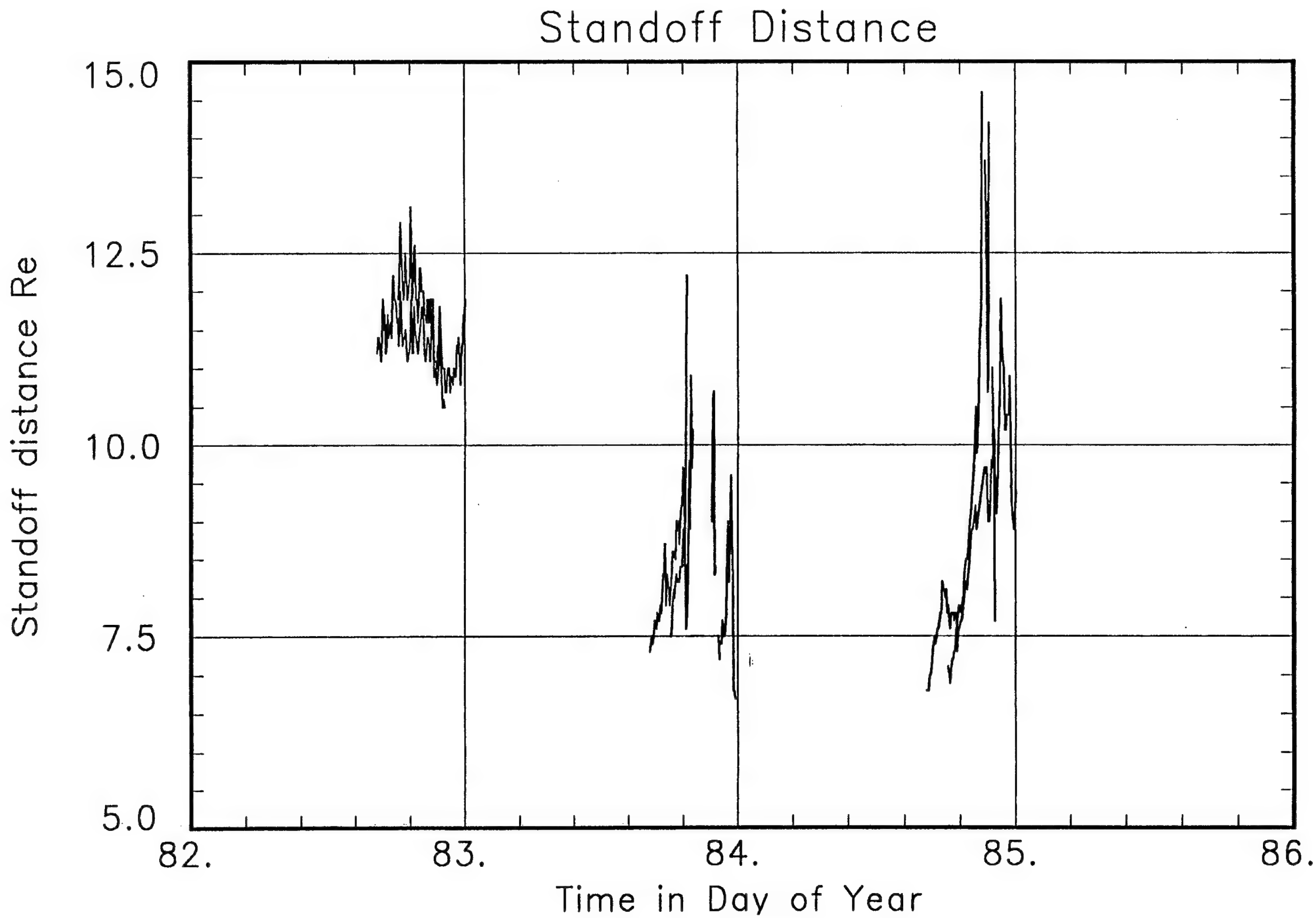


Figure 24. Best guess standoff distance for March 1991 event. Data for important time periods is not available.

5.0 Summary

The main accomplishment of this effort was the development of new tools that are already in extensive use and promise to help develop a real understanding of the magnetosphere and its source and loss mechanisms. The new B,L code is already in extensive use and promises to be one of the most important tools for the future building of magnetospheric radiation belt models. It fully incorporates the ability to work with atmospheric density effects and can be used to hopefully, with the correct data set, develop a low altitude models that correctly tracks the solar cycle.

A powerful new tool for organizing charged particles in a dynamic magnetosphere was developed. The new B,L code was written to allow a time-dependent model to be used to evaluate the first and second invariant. A method was suggested for organizing pitch angle dependent the charged particle fluxes in a dynamic magnetosphere.

A complete time dependent system of codes is presented. In addition to the time-dependent magnetic field and a time dependent B,L code, a time dependent induction electric field code is made available. This code is modularly written and can be used by various researchers to study the induction electric field effects during magnetic disturbances in the magnetosphere.

Early in this program we had great hopes in providing a complete treatise on the morphology of the creation of a new radiation belt. It became the rapture of the particle acceleration phenomena. Our initial enthusiasm was greatly tempered by the fact there was always a hint that it should work and yet when the actual

numbers were run there was always something wrong. One of the great concerns of others is that we are not working in a vacuum magnetosphere and thus the simplistic approach of using the induction field may just not be good enough. We attempted to use some of the wave analyses developed for the Office of Naval Research but found the technique to be cumbersome and difficult. We also found that we were unable to come up with a plausible defensible wave form that we could allow to propagate through the inner magnetosphere during this event. The March 1995 event had a magnetospheric compression rate that was almost the same as the Alfven speed and thus it is expected that there may be large differences between the vacuum equations and the actual delta B and E field changes with in the magnetosphere.

Much needs to be done to successfully address and understand the dynamics of the inner radiation belt. The CRRES project has greatly expanded our understanding of the inner magnetosphere, it has also demonstrated the importance of the inner magnetosphere in understanding the dynamics of the overall system.

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Appendix A

Internal Magnetic Field Subroutines

Subroutine SPIGRF is a fast version of the IGRF internal field subroutine. Instead of using DO loops to expand the spherical harmonic coefficients, it writes out the expansion according to a routine developed by J. Cain. This version of the routine developed for the CRRES program is not designed for stand alone use and is designed to be a part of a total magnetic field program that includes the internal as well as the external magnetic field. The calling arguments are thus passed in a common block and the geomagnetic latitude and longitude are passed via their sines and cosines. This techniques saves computer time. A stand-alone version would be much easier to use if the calling variables were transmitted as standard subroutine arguments. If an easy to use stand alone version is needed, a simple change to the first few lines of the code can produce a very efficient stand-alone internal field code.

The method of coding the magnetic field in SPIGRF is inherently faster than a DO loop version. Furthermore, the addition of a term dropping algorithm increases the speed as much as a factor of 35. The variable CONA dimensioned 11 contains the altitudes at which successive terms are to be dropped. A linear interpolation between these distance values drops the terms off smoothly. The smooth feature is important since it prevents discontinuities in the magnetic field from disrupting the integration steps and the interpolation algorithms in the field line tracing program.

Calling Sequence

The transfer of information between SPIGRF and the calling routine is performed via labeled COMMON GCOM.

INPUT values

YEARI Contains the year for which the coefficients are to be determined. The supplied coefficients are valid from 1945 to the present. The 1945 coefficients are used for years earlier than 1945. Predicting far into the future is hazardous since the time derivative terms do not have long term validity. If YEARI is changed by .1 years since the last call a new updated set of coefficients is calculated. It is suggested that YEARI be used to set the desired epoch and then left constant.

NMAXN Contains the number of terms desired. If this is left 0 then the full 11 term expansion of IGRF is used. If NMAXN is between 2 and 10, then the maximum number of terms is set to that number. Term dropping still takes place for larger distances.

ST Sine of the geographic **co-latitude**

CT Cosine of the geographic **co-latitude**

SPH Sine of the geographic longitude.

CPH Cosine of the geographic longitude

AOR 6371.2/R, where R is the distance from the center of the Earth in km

OUTPUT values

BR The radial component of the magnetic field in gauss (ie. nanotesla)

BT Theta component (south pointing) component

BP Phi component (East)

Each time the field coefficients are updated, the value of the new dipole moment is stored in labeled COMMON /MOMENT/ XM. It is thus available for use by any routine that needs it, such as the L value program.

NOTE: SPIGRF uses a true spherical coordinate system with the z axis along the geographic north pole, the x axis through the longitude of Greenwich. R, Theta and Phi are the true spherical polar coordinates.

The routine will read several of the IGRF Coefficient sets. The coefficient sets are listed at the end of this appendix. Subroutine FLDCOF sets the FORTRAN logical unit for reading the coefficients to 11. The actual read statement for reading the coefficients are found in subroutine GETGAU.

```
      SUBROUTINE SPIGRF
C
C     VERSION 4/91
C     WRITTEN BY K.A. PFITZER  (714) 896-3231
C     SPIGRF IS A MODIFIED VERSION OF J.C. CAIN'S 14 TERM FAST SPHRC
C     ROUTINE.
C     IT HAS BEEN SHORTENED TO 11 TERMS FOR CONSISTENCY WITH THE IGRF
C     COEFFICIENT SET.
C     IT HAS A TRUNCATION FOR LARGE R - THE TRUNCATION BETWEEN TERMS
C     IS SMOOTH AND MAINTAINS AN ACCURACY OVER THE NON-TRUNCATED VERSION
C     OF BETTER THAN .1 NANOTESLA.
C     DEPENDING ON ALTITUDE THIS VERSION RUNS FROM 1.5 TO 35.0 TIMES AS FAST
C     AS THE STANDARD SCHMITT NORMLIZED IGRF ROUTINES
C     THE SUPPORT ROUTINES READ THE STANDARD IGRF COEFFICIENTS AND
C     CONVERT THEM TO GAUSS NORMALIZED FOR USE BY THIS ROUTINE
C
C
C     The first time the routine is called, the routine calls routine
C     call routine FLDCOF to obtain the correct IGRF coefficients.  If
C     the date changes by more than .1 year the coefficients are updated,
C     new coefficients are obtained if required.
C
C     INPUT -- COMMON BLOCK GCOM
C        YEARI    IS THE YEAR, IF YEARI CHANGES, THE COEFFICIENTS ARE
C                 UPDATED.
C        ST       SINE OF THE GEOGRAPHIC CO-LATITUDE.
C        CT       COSINE OF THE GEOGRAPHIC CO-LATITUDE.
C        SPH      SINE OF THE GEOGRAPHIC LONGITUDE.
C        CPH      COSINE OF THE GEOGRAPHIC LONGITUDE.
C        AOR      6371.2/R, WHERE R IS THE GEOCENTRIC DISTANCE IN KM FROM
C                 THE CENTER OF THE EARTH.
C        NMAXN    MAXIMUM NUMBER OF TERMS TO BE USED  (MUST BE LESS OR
C                 EQUAL TO 11). THIS ROUTINE PRESETS IT TO 11
C                 NMAXN OF 11 CORRESPONDS TO THE 10TH ORDER IGRF MODELS
C                 IF NMAXN IS >2 AND <11, NMAXN TERMS ARE USED, ELSE THE
C                 NUMBER OF TERMS USED IS 11 OR THE MAXIMUM TERMS IN THE
C                 IGRF DATA SET.
C     OUTPUT -- COMMON BLOCK GCOM
C        BR       RADIAL COMPONENT OF FIELD IN GAUSS.
C        BT       THETA COMPONENT (SOUTH POINTING) COMPONENT.
C        BP       PHI COMPONENT (EAST)
C
      DIMENSION G(11,11),CONST(11,11),FM(11),FN(11)
      COMMON /MODEL/G
      COMMON /GCOM/ ST,CT,SPH,CPH,AOR,BT,BP,BR,NMAXN,YEARI
      COMMON /MOMENT/XM
      DIMENSION CONA(11)
      DATA YRLAST /-12345./
      DATA IFIRST/0/
      DATA CONA/0.,12.0,8.0,6.0,5.0,4.0,3.2,2.5,2.0,1.6,1.4/
C
C     SET UP INITIAL CONSTANTS DURING FIRST CALL
      IF(IFIRST.NE.0) GO TO 199
      IFIRST=1
      FM(1)=0
      DO 6 N=2,11
      FM(N)=N-1
      FN(N)=N
      DO 6 M=1,N
 6    CONST(N,M)=FLOAT((N-2)**2-(M-1)**2)/FLOAT((2*N-3)*(2*N-5))
```

```
C
C     SET UP THE COEFFICIENTS
C     IF YEARI HAS CHANGED BY MORE THAN .1 YEAR UPDATE THE COEFFICIENTS
C
199   IF(ABS(YRLAST-YEARI).LT.0.1) GO TO 230
      CALL FLDCOF(YEARI,DIMO,MAXN)
      XM=DIMO/1.0E5
      YRLAST=YEARI
C
 230  NMAX=MAXN
      IF(NMAXN.GE.2.AND.NMAXN.LT.MAXN)NMAX=NMAXN
      AR=AOR*AOR*AOR                                                    1-00006
      C2=G(2,2)*CPH+G(1,2)*SPH                                          1-00007
      BR=-(AR+AR)*(G(2,1)*CT+C2*ST)                                     1-00008
      BT=AR*(C2*CT-G(2,1)*ST)                                           1-00009
      BP=AR*(G(1,2)*CPH-G(2,2)*SPH)                                     1-00010
      IF (NMAX.LE.2) RETURN                                             1-00011
      R=1./AOR
      IF(R.GT.CONA(2)) RETURN
      CON=0.
      SP2=SPH                                                           1-00012
      CP2=CPH                                                           1-00013
      P21=CT                                                            1-00014
      P22=ST                                                            1-00015
      DP21=-ST                                                          1-00016
      DP22=CT                                                           1-00017
      N=3
      SP3=(SP2+SP2)*CP2                                                 1-00019
      CP3=(CP2+SP2)*(CP2-SP2)                                           1-00020
      P31=CT*P21-CONST(3,1)                                             1-00021
      P32=CT*P22                                                        1-00022
      P33=ST*P22                                                        1-00023
      DP31=-P32-P32                                                     1-00024
      DP32=CT*DP22-P33                                                  1-00025
      DP33=-DP31                                                        1-00026
      C2=G(3,2)*CP2+G(1,3)*SP2                                          1-00027
      C3=G(3,3)*CP3+G(2,3)*SP3                                          1-00028
      AR=AOR*AR                                                         1-00029
      XR=BR-FN(3)*AR*(G(3,1)*P31+C2*P32+C3*P33)                         1-00030
      XT=BT+AR*(G(3,1)*DP31+C2*DP32+C3*DP33)                            1-00031
      XP=BP-AR*(FM(2)*(G(3,2)*SP2-G(1,3)*CP2)*P21+FM(3)*(G(3,3)*SP3-G(2,1-00032
     +3)*CP3)*P22)                                                      1-00033
      BP=BP*ST                                                          1-00035
      XP=XP*ST                                                          1-00035
      IF(NMAX.LE.3) GO TO 21
      IF(R.GT.CONA(3)) GO TO 20
      N=4
      SP4=SPH*CP3+CPH*SP3                                               1-00037
      CP4=CPH*CP3-SPH*SP3                                               1-00038
      P41=CT*P31-CONST(4,1)*P21                                         1-00039
      DP41=CT*DP31-ST*P31-CONST(4,1)*DP21                               1-00040
      P42=CT*P32-CONST(4,2)*P22                                         1-00041
      DP42=CT*DP32-ST*P32-CONST(4,2)*DP22                               1-00042
      P43=CT*P33                                                        1-00043
      DP43=CT*DP33-ST*P33                                               1-00044
      P44=ST*P33                                                        1-00045
      DP44=FM(4)*P43                                                    1-00046
      C2=G(4,2)*CP2+G(1,4)*SP2                                          1-00047
      C3=G(4,3)*CP3+G(2,4)*SP3                                          1-00048
      C4=G(4,4)*CP4+G(3,4)*SP4                                          1-00049
```

```
      AR=AOR*AR
      BR=XR-FN(4)*AR*(G(4,1)*P41+C2*P42+C3*P43+C4*P44)
      BT=XT+AR*(G(4,1)*DP41+C2*DP42+C3*DP43+C4*DP44)
      BP=XP-AR*(FM(2)*(G(4,2)*SP2-G(1,4)*CP2)*P42+FM(3)*(G(4,3)*SP3-G(2,
     +4)*CP3)*P43+FM(4)*(G(4,4)*SP4-G(3,4)*CP4)*P44)
      IF(NMAX.LE.4) GO TO 11
      IF(R.GT.CONA(4)) GO TO 10
      N=5
      SP5=(SP3+SP3)*CP3
      CP5=(CP3+SP3)*(CP3-SP3)
      P51=CT*P41-CONST(5,1)*P31
      DP51=CT*DP41-ST*P41-CONST(5,1)*DP31
      P52=CT*P42-CONST(5,2)*P32
      DP52=CT*DP42-ST*P42-CONST(5,2)*DP32
      P53=CT*P43-CONST(5,3)*P33
      DP53=CT*DP43-ST*P43-CONST(5,3)*DP33
      P54=CT*P44
      DP54=CT*DP44-ST*P44
      P55=ST*P44
      DP55=FM(5)*P54
      C2=G(5,2)*CP2+G(1,5)*SP2
      C3=G(5,3)*CP3+G(2,5)*SP3
      C4=G(5,4)*CP4+G(3,5)*SP4
      C5=G(5,5)*CP5+G(4,5)*SP5
      AR=AOR*AR
      XR=BR-FN(5)*AR*(G(5,1)*P51+C2*P52+C3*P53+C4*P54+C5*P55)
      XT=BT+AR*(G(5,1)*DP51+C2*DP52+C3*DP53+C4*DP54+C5*DP55)
      XP=BP-AR*(FM(2)*(G(5,2)*SP2-G(1,5)*CP2)*P52+FM(3)*(G(5,3)*SP3-G(2,
     +5)*CP3)*P53+FM(4)*(G(5,4)*SP4-G(3,5)*CP4)*P54+FM(5)*(G(5,5)*SP5-G(
     +4,5)*CP5)*P55)
      IF(NMAX.LE.5) GO TO 21
      IF(R.GT.CONA(5)) GO TO 20
      N=6
      SP6=SPH*CP5+CPH*SP5
      CP6=CPH*CP5-SPH*SP5
      P61=CT*P51-CONST(6,1)*P41
      DP61=CT*DP51-ST*P51-CONST(6,1)*DP41
      P62=CT*P52-CONST(6,2)*P42
      DP62=CT*DP52-ST*P52-CONST(6,2)*DP42
      P63=CT*P53-CONST(6,3)*P43
      DP63=CT*DP53-ST*P53-CONST(6,3)*DP43
      P64=CT*P54-CONST(6,4)*P44
      DP64=CT*DP54-ST*P54-CONST(6,4)*DP44
      P65=CT*P55
      DP65=CT*DP55-ST*P55
      P66=ST*P55
      DP66=FM(6)*P65
      C2=G(6,2)*CP2+G(1,6)*SP2
      C3=G(6,3)*CP3+G(2,6)*SP3
      C4=G(6,4)*CP4+G(3,6)*SP4
      C5=G(6,5)*CP5+G(4,6)*SP5
      C6=G(6,6)*CP6+G(5,6)*SP6
      AR=AOR*AR
      BR=XR-FN(6)*AR*(G(6,1)*P61+C2*P62+C3*P63+C4*P64+C5*P65+C6*P66)
      BT=XT+AR*(G(6,1)*DP61+C2*DP62+C3*DP63+C4*DP64+C5*DP65+C6*DP66)
      BP=XP-AR*(FM(2)*(G(6,2)*SP2-G(1,6)*CP2)*P62+FM(3)*(G(6,3)*SP3-G(2,
     +6)*CP3)*P63+FM(4)*(G(6,4)*SP4-G(3,6)*CP4)*P64+FM(5)*(G(6,5)*SP5-G(
     +4,6)*CP5)*P65+FM(6)*(G(6,6)*SP6-G(5,6)*CP6)*P66)
      IF(NMAX.LE.6) GO TO 11
      IF(R.GT.CONA(6)) GO TO 10
```

```
      N=7
      SP7=(SP4+SP4)*CP4
      CP7=(CP4+SP4)*(CP4-SP4)
      P71=CT*P61-CONST(7,1)*P51
      DP71=CT*DP61-ST*P61-CONST(7,1)*DP51
      P72=CT*P62-CONST(7,2)*P52
      DP72=CT*DP62-ST*P62-CONST(7,2)*DP52
      P73=CT*P63-CONST(7,3)*P53
      DP73=CT*DP63-ST*P63-CONST(7,3)*DP53
      P74=CT*P64-CONST(7,4)*P54
      DP74=CT*DP64-ST*P64-CONST(7,4)*DP54
      P75=CT*P65-CONST(7,5)*P55
      DP75=CT*DP65-ST*P65-CONST(7,5)*DP55
      P76=CT*P66
      DP76=CT*DP66-ST*P66
      P77=ST*P66
      DP77=FM(7)*P76
      C2=G(7,2)*CP2+G(1,7)*SP2
      C3=G(7,3)*CP3+G(2,7)*SP3
      C4=G(7,4)*CP4+G(3,7)*SP4
      C5=G(7,5)*CP5+G(4,7)*SP5
      C6=G(7,6)*CP6+G(5,7)*SP6
      C7=G(7,7)*CP7+G(6,7)*SP7
      AR=AOR*AR
      XR=BR-FN(7)*AR*(G(7,1)*P71+C2*P72+C3*P73+C4*P74+C5*P75+C6*P76+C7*P
     +77)
      XT=BT+AR*(G(7,1)*DP71+C2*DP72+C3*DP73+C4*DP74+C5*DP75+C6*DP76+C7*D
     +P77)
      XP=BP-AR*(FM(2)*(G(7,2)*SP2-G(1,7)*CP2)*P72+FM(3)*(G(7,3)*SP3-G(2,
     +7)*CP3)*P73+FM(4)*(G(7,4)*SP4-G(3,7)*CP4)*P74+FM(5)*(G(7,5)*SP5-G(
     +4,7)*CP5)*P75+FM(6)*(G(7,6)*SP6-G(5,7)*CP6)*P76+FM(7)*(G(7,7)*SP7-
     +G(6,7)*CP7)*P77)
      IF(NMAX.LE.7) GO TO 21
      IF(R.GT.CONA(7)) GO TO 20
      N=8
      SP8=SPH*CP7+CPH*SP7
      CP8=CPH*CP7-SPH*SP7
      P81=CT*P71-CONST(8,1)*P61
      DP81=CT*DP71-ST*P71-CONST(8,1)*DP61
      P82=CT*P72-CONST(8,2)*P62
      DP82=CT*DP72-ST*P72-CONST(8,2)*DP62
      P83=CT*P73-CONST(8,3)*P63
      DP83=CT*DP73-ST*P73-CONST(8,3)*DP63
      P84=CT*P74-CONST(8,4)*P64
      DP84=CT*DP74-ST*P74-CONST(8,4)*DP64
      P85=CT*P75-CONST(8,5)*P65
      DP85=CT*DP75-ST*P75-CONST(8,5)*DP65
      P86=CT*P76-CONST(8,6)*P66
      DP86=CT*DP76-ST*P76-CONST(8,6)*DP66
      P87=CT*P77
      DP87=CT*DP77-ST*P77
      P88=ST*P77
      DP88=FM(8)*P87
      C2=G(8,2)*CP2+G(1,8)*SP2
      C3=G(8,3)*CP3+G(2,8)*SP3
      C4=G(8,4)*CP4+G(3,8)*SP4
      C5=G(8,5)*CP5+G(4,8)*SP5
      C6=G(8,6)*CP6+G(5,8)*SP6
      C7=G(8,7)*CP7+G(6,8)*SP7
      C8=G(8,8)*CP8+G(7,8)*SP8
```

```
      AR=AOR*AR                                                         1-00166
      BR=XR-FN(8)*AR*(G(8,1)*P81+C2*P82+C3*P83+C4*P84+C5*P85+C6*P86+C7*P1-00167
     +87+C8*P88)                                                        1-00168
      BT=XT+AR*(G(8,1)*DP81+C2*DP82+C3*DP83+C4*DP84+C5*DP85+C6*DP86+C7*D1-00169
     +P87+C8*DP88)                                                      1-00170
      BP=XP-AR*(FM(2)*(G(8,2)*SP2-G(1,8)*CP2)*P82+FM(3)*(G(8,3)*SP3-G(2,1-00171
     +8)*CP3)*P83+FM(4)*(G(8,4)*SP4-G(3,8)*CP4)*P84+FM(5)*(G(8,5)*SP5-G(1-00172
     +4,8)*CP5)*P85+FM(6)*(G(8,6)*SP6-G(5,8)*CP6)*P86+FM(7)*(G(8,7)*SP7-1-00173
     +G(6,8)*CP7)*P87+FM(8)*(G(8,8)*SP8-G(7,8)*CP8)*P88)                1-00174
      IF(NMAX.LE.8) GO TO 11
      IF(R.GT.CONA(8)) GO TO 10
      N=9
      SP9=(SP5+SP5)*CP5                                                 1-00177
      CP9=(CP5+SP5)*(CP5-SP5)                                           1-00178
      P91=CT*P81-CONST(9,1)*P71                                         1-00179
      DP91=CT*DP81-ST*P81-CONST(9,1)*DP71                               1-00180
      P92=CT*P82-CONST(9,2)*P72                                         1-00181
      DP92=CT*DP82-ST*P82-CONST(9,2)*DP72                               1-00182
      P93=CT*P83-CONST(9,3)*P73                                         1-00183
      DP93=CT*DP83-ST*P83-CONST(9,3)*DP73                               1-00184
      P94=CT*P84-CONST(9,4)*P74                                         1-00185
      DP94=CT*DP84-ST*P84-CONST(9,4)*DP74                               1-00186
      P95=CT*P85-CONST(9,5)*P75                                         1-00187
      DP95=CT*DP85-ST*P85-CONST(9,5)*DP75                               1-00188
      P96=CT*P86-CONST(9,6)*P76                                         1-00189
      DP96=CT*DP86-ST*P86-CONST(9,6)*DP76                               1-00190
      P97=CT*P87-CONST(9,7)*P77                                         1-00191
      DP97=CT*DP87-ST*P87-CONST(9,7)*DP77                               1-00192
      P98=CT*P88                                                        1-00193
      DP98=CT*DP88-ST*P88                                               1-00194
      P99=ST*P88                                                        1-00195
      DP99=FM(9)*P98                                                    1-00196
      C2=G(9,2)*CP2+G(1,9)*SP2                                          1-00197
      C3=G(9,3)*CP3+G(2,9)*SP3                                          1-00198
      C4=G(9,4)*CP4+G(3,9)*SP4                                          1-00199
      C5=G(9,5)*CP5+G(4,9)*SP5                                          1-00200
      C6=G(9,6)*CP6+G(5,9)*SP6                                          1-00201
      C7=G(9,7)*CP7+G(6,9)*SP7                                          1-00202
      C8=G(9,8)*CP8+G(7,9)*SP8                                          1-00203
      C9=G(9,9)*CP9+G(8,9)*SP9                                          1-00204
      AR=AOR*AR                                                         1-00205
      XR=BR-FN(9)*AR*(G(9,1)*P91+C2*P92+C3*P93+C4*P94+C5*P95+C6*P96+C7*P1-00206
     +97+C8*P98+C9*P99)                                                 1-00207
      XT=BT+AR*(G(9,1)*DP91+C2*DP92+C3*DP93+C4*DP94+C5*DP95+C6*DP96+C7*D1-00208
     +P97+C8*DP98+C9*DP99)                                              1-00209
      XP=BP-AR*(FM(2)*(G(9,2)*SP2-G(1,9)*CP2)*P92+FM(3)*(G(9,3)*SP3-G(2,1-00210
     +9)*CP3)*P93+FM(4)*(G(9,4)*SP4-G(3,9)*CP4)*P94+FM(5)*(G(9,5)*SP5-G(1-00211
     +4,9)*CP5)*P95+FM(6)*(G(9,6)*SP6-G(5,9)*CP6)*P96+FM(7)*(G(9,7)*SP7-1-00212
     +G(6,9)*CP7)*P97+FM(8)*(G(9,8)*SP8-G(7,9)*CP8)*P98+FM(9)*(G(9,9)*SP1-00213
     +9-G(8,9)*CP9)*P99)                                                1-00214
      IF(NMAX.LE.9) GO TO 21
      IF(R.GT.CONA(9)) GO TO 20
      N=10
      SP10=SPH*CP9+CPH*SP9                                              1-00217
      CP10=CPH*CP9-SPH*SP9                                              1-00218
      P101=CT*P91-CONST(10,1)*P81                                       1-00219
      DP101=CT*DP91-ST*P91-CONST(10,1)*DP81                             1-00220
      P102=CT*P92-CONST(10,2)*P82                                       1-00221
      DP102=CT*DP92-ST*P92-CONST(10,2)*DP82                             1-00222
      P103=CT*P93-CONST(10,3)*P83                                       1-00223
```

```
      DP103=CT*DP93-ST*P93-CONST(10,3)*DP83
      P104=CT*P94-CONST(10,4)*P84
      DP104=CT*DP94-ST*P94-CONST(10,4)*DP84
      P105=CT*P95-CONST(10,5)*P85
      DP105=CT*DP95-ST*P95-CONST(10,5)*DP85
      P106=CT*P96-CONST(10,6)*P86
      DP106=CT*DP96-ST*P96-CONST(10,6)*DP86
      P107=CT*P97-CONST(10,7)*P87
      DP107=CT*DP97-ST*P97-CONST(10,7)*DP87
      P108=CT*P98-CONST(10,8)*P88
      DP108=CT*DP98-ST*P98-CONST(10,8)*DP88
      P109=CT*P99
      DP109=CT*DP99-ST*P99
      P1010=ST*P99
      DP1010=FM(10)*P109
      C2=G(10,2)*CP2+G(1,10)*SP2
      C3=G(10,3)*CP3+G(2,10)*SP3
      C4=G(10,4)*CP4+G(3,10)*SP4
      C5=G(10,5)*CP5+G(4,10)*SP5
      C6=G(10,6)*CP6+G(5,10)*SP6
      C7=G(10,7)*CP7+G(6,10)*SP7
      C8=G(10,8)*CP8+G(7,10)*SP8
      C9=G(10,9)*CP9+G(8,10)*SP9
      C10=G(10,10)*CP10+G(9,10)*SP10
      AR=AOR*AR
      BR=XR-FN(10)*AR*(G(10,1)*P101+C2*P102+C3*P103+C4*P104+C5*P105+C6*P1
     +106+C7*P107+C8*P108+C9*P109+C10*P1010)
      BT=XT+AR*(G(10,1)*DP101+C2*DP102+C3*DP103+C4*DP104+C5*DP105+C6*DP1
     +06+C7*DP107+C8*DP108+C9*DP109+C10*DP1010)
      BP=XP-AR*(FM(2)*(G(10,2)*SP2-G(1,10)*CP2)*P102+FM(3)*(G(10,3)*SP3-
     +G(2,10)*CP3)*P103+FM(4)*(G(10,4)*SP4-G(3,10)*CP4)*P104+FM(5)*(G(10
     +,5)*SP5-G(4,10)*CP5)*P105+FM(6)*(G(10,6)*SP6-G(5,10)*CP6)*P106+FM(
     +7)*(G(10,7)*SP7-G(6,10)*CP7)*P107+FM(8)*(G(10,8)*SP8-G(7,10)*CP8)*
     +P108+FM(9)*(G(10,9)*SP9-G(8,10)*CP9)*P109+FM(10)*(G(10,10)*SP10-G(
     +9,10)*CP10)*P1010)
      IF(NMAX.LE.10) GO TO 11
      IF(R.GT.CONA(10)) GO TO 10
      N=11
      SP11=(SP6+SP6)*CP6
      CP11=(CP6+SP6)*(CP6-SP6)
      P111=CT*P101-CONST(11,1)*P91
      DP111=CT*DP101-ST*P101-CONST(11,1)*DP91
      P112=CT*P102-CONST(11,2)*P92
      DP112=CT*DP102-ST*P102-CONST(11,2)*DP92
      P113=CT*P103-CONST(11,3)*P93
      DP113=CT*DP103-ST*P103-CONST(11,3)*DP93
      P114=CT*P104-CONST(11,4)*P94
      DP114=CT*DP104-ST*P104-CONST(11,4)*DP94
      P115=CT*P105-CONST(11,5)*P95
      DP115=CT*DP105-ST*P105-CONST(11,5)*DP95
      P116=CT*P106-CONST(11,6)*P96
      DP116=CT*DP106-ST*P106-CONST(11,6)*DP96
      P117=CT*P107-CONST(11,7)*P97
      DP117=CT*DP107-ST*P107-CONST(11,7)*DP97
      P118=CT*P108-CONST(11,8)*P98
      DP118=CT*DP108-ST*P108-CONST(11,8)*DP98
      P119=CT*P109-CONST(11,9)*P99
      DP119=CT*DP109-ST*P109-CONST(11,9)*DP99
      P1110=CT*P1010
      DP1110=CT*DP1010-ST*P1010
```

```
      P1111=ST*P1010                                                    1-00283
      DP1111=FM(11)*P1110                                               1-00284
      C2=G(11,2)*CP2+G(1,11)*SP2                                        1-00285
      C3=G(11,3)*CP3+G(2,11)*SP3                                        1-00286
      C4=G(11,4)*CP4+G(3,11)*SP4                                        1-00287
      C5=G(11,5)*CP5+G(4,11)*SP5                                        1-00288
      C6=G(11,6)*CP6+G(5,11)*SP6                                        1-00289
      C7=G(11,7)*CP7+G(6,11)*SP7                                        1-00290
      C8=G(11,8)*CP8+G(7,11)*SP8                                        1-00291
      C9=G(11,9)*CP9+G(8,11)*SP9                                        1-00292
      C10=G(11,10)*CP10+G(9,11)*SP10                                    1-00293
      C11=G(11,11)*CP11+G(10,11)*SP11                                   1-00294
      AR=AOR*AR                                                         1-00295
      BR=BR-FN(11)*AR*(G(11,1)*P111+C2*P112+C3*P113+C4*P114+C5*P115+C6*P1-00296
     +116+C7*P117+C8*P118+C9*P119+C10*P1110+C11*P1111)                  1-00297
      BT=BT+AR*(G(11,1)*DP111+C2*DP112+C3*DP113+C4*DP114+C5*DP115+C6*DP11-00298
     +16+C7*DP117+C8*DP118+C9*DP119+C10*DP1110+C11*DP1111)              1-00299
      BP=BP-AR*(FM(2)*(G(11,2)*SP2-G(1,11)*CP2)*P112+FM(3)*(G(11,3)*SP3-1-00300
     +G(2,11)*CP3)*P113+FM(4)*(G(11,4)*SP4-G(3,11)*CP4)*P114+FM(5)*(G(111-00301
     +,5)*SP5-G(4,11)*CP5)*P115+FM(6)*(G(11,6)*SP6-G(5,11)*CP6)*P116+FM(1-00302
     +7)*(G(11,7)*SP7-G(6,11)*CP7)*P117+FM(8)*(G(11,8)*SP8-G(7,11)*CP8)*1-00303
     +P118+FM(9)*(G(11,9)*SP9-G(8,11)*CP9)*P119+FM(10)*(G(11,10)*SP10-G(1-00304
     +9,11)*CP10)*P1110+FM(11)*(G(11,11)*SP11-G(10,11)*CP11)*P1111)     1-00305
      BP=BP/ST
      IF (NMAX.LE.11) RETURN                                            1-00306
      WRITE (*,2) NMAX                                                  1-00476
      RETURN                                                            1-00477
C
    2 FORMAT(57H0 ERROR, THIS MODEL ONLY FOR NMAX=<11, CALL WAS FOR NMAX
     *=,I5)
C     MAKE A SMOOTH FIT BETWEEN TRUNCATED TERMS.
   10 CON=(R-CONA(N))/(CONA(N-1)-CONA(N))
   11 BR=BR+(XR-BR)*CON
      BT=BT+(XT-BT)*CON
      BP=(BP+(XP-BP)*CON)/ST
      RETURN
   20 CON=(R-CONA(N))/(CONA(N-1)-CONA(N))
   21 BR=XR+(BR-XR)*CON
      BT=XT+(BT-XT)*CON
      BP=(XP+(BP-XP)*CON)/ST
      RETURN
      END                                                               1-00480
C
```

```
      SUBROUTINE FLDCOF(YEAR,DIMO,NMAXI)
C-----------------------------------------------------------------------
C  DETERMINES COEFFICIENTS AND DIPOL MOMENT FROM IGRF MODELS
C
C     INPUT:  YEAR    DECIMAL YEAR FOR WHICH GEOMAGNETIC FIELD IS TO
C                     BE CALCULATED
C     OUTPUT: DIMO    GEOMAGNETIC DIPOL MOMENT IN GAUSS (NORMALIZED
C                     TO EARTH'S RADIUS) AT THE TIME (YEAR)
C  THIS ROUNTINE WAS INITIALLY WRITTEN BY
C  D. BILITZA, NSSDC, GSFC, CODE 633, GREENBELT, MD 20771,
C     (301)286-9536   NOV 1987.
C  MODIFIED BY K. A. PFITZER MDSSC TO WORK WITH GAUSS NORMALIZED COEFF.
C-----------------------------------------------------------------------
      CHARACTER*19          FILMOD, FIL1, FIL2
      DIMENSION      GH1(11,11), GH2(11,11),
     1                      DTEMOD(12),FILMOD(12)
      DOUBLE PRECISION      F0
      COMMON/MODEL/         G(11,11)
      COMMON/GENER/         UMR,ERAD,AQUAD,BQUAD
      DATA           FILMOD /'dgrf45.dat', 'dgrf50.dat',
     1              'dgrf55.dat', 'dgrf60.dat',
     *              'dgrf65.dat',
     2              'dgrf70.dat', 'dgrf75.dat',
     *              'dgrf80.dat',
     3              'dgrf85.dat', 'dgrf90.dat',
     *               'igrf95.dat', 'igrf95s.dat'/
      DATA           DTEMOD / 1945., 1950., 1955., 1960.,
     1              1965., 1970., 1975., 1980., 1985., 1990.,
     2              1995., 2000./
      DATA           LOLD/0/
C
      IU = 11
      NUMYE=11
C-- DETERMINE IGRF-YEARS FOR INPUT-YEAR
      TIME = YEAR
      IYEA = INT(YEAR/5.)*5
      L = (IYEA - 1945)/5 + 1
C
      IF (L.NE.LOLD) THEN
      LOLD=L
      IF(L.LT.1) L=1
      IF(L.GT.NUMYE) L=NUMYE
      DTE1 = DTEMOD(L)
      FIL1 = FILMOD(L)
      DTE2 = DTEMOD(L+1)
      FIL2 = FILMOD(L+1)
C-- GET IGRF COEFFICIENTS FOR THE BOUNDARY YEARS
      CALL GETGAU (IU, FIL1, NMAX1, ERAD, GH1, IER)
      IF (IER .NE. 0) THEN
        WRITE (*,101) IU,FIL1,NMAX1,ERAD,IER
101       FORMAT (//'       Error in subroutine FELDCOF'/
     1           ' IU, FIL1, NMAX1, ERAD, IER:'/
     2           I10,A11,I10,1PE12.3,I10)
        STOP
      ENDIF
      CALL GETGAU (IU, FIL2, NMAX2, ERAD, GH2, IER)
        IF (IER .NE. 0) THEN
        WRITE (*,102) IU,FIL2,NMAX2,ERAD,IER
102       FORMAT (//'       Error in subroutine FELDCOF'/
     1           ' IU, FIL2, NMAX2, ERAD, IER:'/
```

```
     2        I10,A19,I10,1PE12.3,I10)
        STOP
      ENDIF
      ENDIF
C-- DETERMINE IGRF COEFFICIENTS FOR YEAR
      IF (L .LE. NUMYE-1) THEN
        CALL CINTRP (YEAR, DTE1, NMAX1, GH1, DTE2,
     1        NMAX2, GH2, NMAXI, G)
      ELSE
        CALL EXTRAP (YEAR, DTE1, NMAX1, GH1, NMAX2,
     1        GH2, NMAXI, G)
      ENDIF
C-- DETERMINE MAGNETIC DIPOL MOMENT
      F0=G(2,1)**2+G(2,2)**2+G(1,2)**2
      DIMO=SQRT(F0)
      RETURN
      END
```

```
      SUBROUTINE GETGAU (IU, FSPEC, NMAX, ERAD, G, IER)

C ===============================================================
C
C     Reads spherical harmonic coefficients from the specified
C     file into an array and converts the coefficients to Gauss
C     normalized coefficients.
C
C     Input:
C         IU       - Logical unit number
C         FSPEC    - File specification
C
C     Output:
C         NMAX     - Maximum degree and order of model
C         ERAD     - Earth's radius associated with the spherical
C                    harmonic coefficients, in the same units as
C                    elevation
C         GH       - Gauss quasi-normal internal spherical
C                    harmonic coefficients
C         IER      - Error number: =  0, no error
C                                  = -2, records out of order
C                                  = FORTRAN run-time error number
C
C ===============================================================

      CHARACTER    FSPEC*(*)
      DIMENSION    G(11,11)

C ---------------------------------------------------------------
C     Open coefficient file. Read past first header record.
C     Read degree and order of model and Earth's radius.
C ---------------------------------------------------------------
      OPEN (IU, FILE=FSPEC, STATUS='OLD', IOSTAT=IER, ERR=999)
C     1   READONLY)

      DO 10 I=1,11
      DO 10 J=1,11
10    G(I,J)=0.

      READ (IU, *, IOSTAT=IER, ERR=999)
      READ (IU, *, IOSTAT=IER, ERR=999) MAXN, ERAD

      IF(MAXN.GT.10)MAXN=10
      DO 30 NN=1,MAXN
      DO 20 MM=0,NN
      READ (IU, *, IOSTAT=IER, ERR=999) LN, LM, GNM, HNM
      IF(NN.NE.LN.OR.MM.NE.LM)THEN
       IER=-2
       GOTO 999
      ENDIF

      N=LN+1
      M=LM+1
      G(N,M)=GNM
      IF(LM.EQ.0) goto 20
      G(LM,N)=HNM
20    CONTINUE
30    CONTINUE
      NMAX=MAXN+1
```

```
C    Convert to Gauss normalized
       DO 55 N=1,NMAX
       DO 55 M=1,NMAX
       CALL CONVRT(g(n,m),n,m,1)
55     CONTINUE

999    CLOSE (IU)
       RETURN
       END
```

```
      SUBROUTINE CONVRT(G,I,L,K)
      DIMENSION S(11,11)
      LOGICAL NEXT
      DATA NEXT/.FALSE./
      IF (NEXT) GOTO 2
      NEXT=.TRUE.
      S(1,1)=-1.
      DO 1 N=2,11
      S(N,1)=S(N-1,1)*FLOAT(2*N-3)/FLOAT(N-1)
      S(1,N)=0.
      J=2
      DO 1 M=2,N
      S(N,M)=S(N,M-1)*SQRT((FLOAT(N-M+1)*J)/FLOAT(N+M-2))
      S(M-1,N)=S(N,M)
1     J=1
2     IF(K.GT.1) GOTO 3
      G=G*S(I,L)
      RETURN
3     G=G/S(I,L)
      RETURN
      END
C
C
```

```
      SUBROUTINE CINTRP (DATE, DTE1, NMAX1, GH1, DTE2,
     1                   NMAX2, GH2, NMAX, GH)
C ================================================================
C
C     Interpolates linearly, in time, between two spherical
C     harmonic models.
C
C     Input:
C         DATE      - Date of resulting model (in decimal year)
C         DTE1      - Date of earlier model
C         NMAX1     - Maximum degree and order of earlier model
C         GH1       - Gauss quasi-normal internal spherical
C                     harmonic coefficients of earlier model
C         DTE2      - Date of later model
C         NMAX2     - Maximum degree and order of later model
C         GH2       - Gauss quasi-normal internal spherical
C                     harmonic coefficients of later model
C
C     Output:
C         GH        - Coefficients of resulting model
C         NMAX      - Maximum degree and order of resulting model
C
C ================================================================

      DIMENSION   GH1(11,11), GH2(11,11), GH(11,11)

      NMAX=MAX0(NMAX1,NMAX2)
      FACTOR=(DATE-DTE1)/(DTE2-DTE1)
      DO 234 J=  1,11
      DO 234 I = 1, 11
234   GH(I,J) = GH1(I,J) + FACTOR * (GH2(I,J) - GH1(I,J))

      RETURN
      END
C
C
```

```
      SUBROUTINE EXTRAP (DATE, DTE1, NMAX1, GH1, NMAX2,
     1                   GH2, NMAX, GH)

C ===============================================================
C
C     Extrapolates linearly a spherical harmonic model with a
C     rate-of-change model.
C
C     Input:
C         DATE     - Date of resulting model (in decimal year)
C         DTE1     - Date of base model
C         NMAX1    - Maximum degree and order of base model
C         GH1      - Gauss quasi-normal internal spherical
C                    harmonic coefficients of base model
C         NMAX2    - Maximum degree and order of rate-of-change
C                    model
C         GH2      - Gauss quasi-normal internal spherical
C                    harmonic coefficients of rate-of-change model
C
C     Output:
C         GH       - Coefficients of resulting model
C         NMAX     - Maximum degree and order of resulting model
C
C ===============================================================

      DIMENSION   GH1(11,11), GH2(11,11), GH(11,11)

      NMAX=MAX0(NMAX1,NMAX2)
      FACTOR = (DATE - DTE1)

      DO 567 J=1,11
      DO 567 I = 1,11
567   GH(I,J) = GH1(I,J) + FACTOR * GH2(I,J)

      RETURN
      END
□
```

dgrf45

10 6371.2 1945.0

1	0	-30594.	0.
1	1	-2285.	5810.
2	0	-1244.	0.
2	1	2990.	-1702.
2	2	1578.	477.
3	0	1282.	0.
3	1	-1834.	-499.
3	2	1255.	186.
3	3	913.	-11.
4	0	944.	0.
4	1	776.	144.
4	2	544.	-276.
4	3	-421.	-55.
4	4	304.	-178.
5	0	-253.	0.
5	1	346.	-12.
5	2	194.	95.
5	3	-20.	-67.
5	4	-142.	-119.
5	5	-82.	82.
6	0	59.	0.
6	1	57.	6.
6	2	6.	100.
6	3	-246.	16.
6	4	-25.	-9.
6	5	21.	-16.
6	6	-104.	-39.
7	0	70.	0.
7	1	-40.	-45.
7	2	0.	-18.
7	3	0.	2.
7	4	-29.	6.
7	5	-10.	28.
7	6	15.	-17.
7	7	29.	-22.
8	0	13.	0.
8	1	7.	12.
8	2	-8.	-21.
8	3	-5.	-12.
8	4	9.	-7.
8	5	7.	2.
8	6	-10.	18.
8	7	7.	3.
8	8	2.	-11.
9	0	5.	0.
9	1	-21.	-27.
9	2	1.	17.
9	3	-11.	29.
9	4	3.	-9.
9	5	16.	4.
9	6	-3.	9.
9	7	-4.	6.
9	8	-3.	1.
9	9	-4.	8.
10	0	-3.	0.
10	1	11.	5.
10	2	1.	1.
10	3	2.	-20.
10	4	-5.	-1.
10	5	-1.	-6.
10	6	8.	6.
10	7	-1.	-4.
10	8	-3.	-2.
10	9	5.	0.
10	10	-2.	-2.

dgrf50

10 6371.2 1950.0

1	0	-30554.	0.
1	1	-2250.	5815.
2	0	-1341.	0.
2	1	2998.	-1810.
2	2	1576.	381.
3	0	1297.	0.
3	1	-1889.	-476.
3	2	1274.	206.
3	3	896.	-46.
4	0	954.	0.
4	1	792.	136.
4	2	528.	-278.
4	3	-408.	-37.
4	4	303.	-210.
5	0	-240.	0.
5	1	349.	3.
5	2	211.	103.
5	3	-20.	-87.
5	4	-147.	-122.
5	5	-76.	80.
6	0	54.	0.
6	1	57.	-1.
6	2	4.	99.
6	3	-247.	33.
6	4	-16.	-12.
6	5	12.	-12.
6	6	-105.	-30.
7	0	65.	0.
7	1	-55.	-35.
7	2	2.	-17.
7	3	1.	0.
7	4	-40.	10.
7	5	-7.	36.
7	6	5.	-18.
7	7	19.	-16.
8	0	22.	0.
8	1	15.	5.
8	2	-4.	-22.
8	3	-1.	0.
8	4	11.	-21.
8	5	15.	-8.
8	6	-13.	17.
8	7	5.	-4.
8	8	-1.	-17.
9	0	3.	0.
9	1	-7.	-24.
9	2	-1.	19.
9	3	-25.	12.
9	4	10.	2.
9	5	5.	2.
9	6	-5.	8.
9	7	-2.	8.
9	8	3.	-11.
9	9	8.	-7.
10	0	-8.	0.
10	1	4.	13.
10	2	-1.	-2.
10	3	13.	-10.
10	4	-4.	2.
10	5	4.	-3.
10	6	12.	6.
10	7	3.	-3.
10	8	2.	6.
10	9	10.	11.
10	10	3.	8.

dgrf55

10 6371.2 1955.0

1	0	-30500.	0.
1	1	-2215.	5820.
2	0	-1440.	0.
2	1	3003.	-1898.
2	2	1581.	291.
3	0	1302.	0.
3	1	-1944.	-462.
3	2	1288.	216.
3	3	882.	-83.
4	0	958.	0.
4	1	796.	133.
4	2	510.	-274.
4	3	-397.	-23.
4	4	290.	-230.
5	0	-229.	0.
5	1	360.	15.
5	2	230.	110.
5	3	-23.	-98.
5	4	-152.	-121.
5	5	-69.	78.
6	0	47.	0.
6	1	57.	-9.
6	2	3.	96.
6	3	-247.	48.
6	4	-8.	-16.
6	5	7.	-12.
6	6	-107.	-24.
7	0	65.	0.
7	1	-56.	-50.
7	2	2.	-24.
7	3	10.	-4.
7	4	-32.	8.
7	5	-11.	28.
7	6	9.	-20.
7	7	18.	-18.
8	0	11.	0.
8	1	9.	10.
8	2	-6.	-15.
8	3	-14.	5.
8	4	6.	-23.
8	5	10.	3.
8	6	-7.	23.
8	7	6.	-4.
8	8	9.	-13.
9	0	4.	0.
9	1	9.	-11.
9	2	-4.	12.
9	3	-5.	7.
9	4	2.	6.
9	5	4.	-2.
9	6	1.	10.
9	7	2.	7.
9	8	2.	-6.
9	9	5.	5.
10	0	-3.	0.
10	1	-5.	-4.
10	2	-1.	0.
10	3	2.	-8.
10	4	-3.	-2.
10	5	7.	-4.
10	6	4.	1.
10	7	-2.	-3.
10	8	6.	7.
10	9	-2.	-1.
10	10	0.	-3.

dgrf60

10 6371.2 1960.0

1	0	-30421.	0.
1	1	-2169.	5791.
2	0	-1555.	0.
2	1	3002.	-1967.
2	2	1590.	206.
3	0	1302.	0.
3	1	-1992.	-414.
3	2	1289.	224.
3	3	878.	-130.
4	0	957.	0.
4	1	800.	135.
4	2	504.	-278.
4	3	-394.	3.
4	4	269.	-255.
5	0	-222.	0.
5	1	362.	16.
5	2	242.	125.
5	3	-26.	-117.
5	4	-156.	-114.
5	5	-63.	81.
6	0	46.	0.
6	1	58.	-10.
6	2	1.	99.
6	3	-237.	60.
6	4	-1.	-20.
6	5	-2.	-11.
6	6	-113.	-17.
7	0	67.	0.
7	1	-56.	-55.
7	2	5.	-28.
7	3	15.	-6.
7	4	-32.	7.
7	5	-7.	23.
7	6	17.	-18.
7	7	8.	-17.
8	0	15.	0.
8	1	6.	11.
8	2	-4.	-14.
8	3	-11.	7.
8	4	2.	-18.
8	5	10.	4.
8	6	-5.	23.
8	7	10.	1.
8	8	8.	-20.
9	0	4.	0.
9	1	6.	-18.
9	2	0.	12.
9	3	-9.	2.
9	4	1.	0.
9	5	4.	-3.
9	6	-1.	9.
9	7	-2.	8.
9	8	3.	0.
9	9	-1.	5.
10	0	1.	0.
10	1	-3.	4.
10	2	4.	1.
10	3	0.	0.
10	4	-1.	2.
10	5	4.	-5.
10	6	6.	1.
10	7	1.	-1.
10	8	-1.	6.
10	9	2.	0.
10	10	0.	-7.

dgrf65

10 6371.2 1965.0

1	0	-30334.	0.
1	1	-2119.	5776.
2	0	-1662.	0.
2	1	2997.	-2016.
2	2	1594.	114.
3	0	1297.	0.
3	1	-2038.	-404.
3	2	1292.	240.
3	3	856.	-165.
4	0	957.	0.
4	1	804.	148.
4	2	479.	-269.
4	3	-390.	13.
4	4	252.	-269.
5	0	-219.	0.
5	1	358.	19.
5	2	254.	128.
5	3	-31.	-126.
5	4	-157.	-97.
5	5	-62.	81.
6	0	45.	0.
6	1	61.	-11.
6	2	8.	100.
6	3	-228.	68.
6	4	4.	-32.
6	5	1.	-8.
6	6	-111.	-7.
7	0	75.	0.
7	1	-57.	-61.
7	2	4.	-27.
7	3	13.	-2.
7	4	-26.	6.
7	5	-6.	26.
7	6	13.	-23.
7	7	1.	-12.
8	0	13.	0.
8	1	5.	7.
8	2	-4.	-12.
8	3	-14.	9.
8	4	0.	-16.
8	5	8.	4.
8	6	-1.	24.
8	7	11.	-3.
8	8	4.	-17.
9	0	8.	0.
9	1	10.	-22.
9	2	2.	15.
9	3	-13.	7.
9	4	10.	-4.
9	5	-1.	-5.
9	6	-1.	10.
9	7	5.	10.
9	8	1.	-4.
9	9	-2.	1.
10	0	-2.	0.
10	1	-3.	2.
10	2	2.	1.
10	3	-5.	2.
10	4	-2.	6.
10	5	4.	-4.
10	6	4.	0.
10	7	0.	-2.
10	8	2.	3.
10	9	2.	0.
10	10	0.	-6.

dgrf70

10 6371.2 1970.0

1	0	-30220.	0.
1	1	-2068.	5737.
2	0	-1781.	0.
2	1	3000.	-2047.
2	2	1611.	25.
3	0	1287.	0.
3	1	-2091.	-366.
3	2	1278.	251.
3	3	838.	-196.
4	0	952.	0.
4	1	800.	167.
4	2	461.	-266.
4	3	-395.	26.
4	4	234.	-279.
5	0	-216.	0.
5	1	359.	26.
5	2	262.	139.
5	3	-42.	-139.
5	4	-160.	-91.
5	5	-56.	83.
6	0	43.	0.
6	1	64.	-12.
6	2	15.	100.
6	3	-212.	72.
6	4	2.	-37.
6	5	3.	-6.
6	6	-112.	1.
7	0	72.	0.
7	1	-57.	-70.
7	2	1.	-27.
7	3	14.	-4.
7	4	-22.	8.
7	5	-2.	23.
7	6	13.	-23.
7	7	-2.	-11.
8	0	14.	0.
8	1	6.	7.
8	2	-2.	-15.
8	3	-13.	6.
8	4	-3.	-17.
8	5	5.	6.
8	6	0.	21.
8	7	11.	-6.
8	8	3.	-16.
9	0	8.	0.
9	1	10.	-21.
9	2	2.	16.
9	3	-12.	6.
9	4	10.	-4.
9	5	-1.	-5.
9	6	0.	10.
9	7	3.	11.
9	8	1.	-2.
9	9	-1.	1.
10	0	-3.	0.
10	1	-3.	1.
10	2	2.	1.
10	3	-5.	3.
10	4	-1.	4.
10	5	6.	-4.
10	6	4.	0.
10	7	1.	-1.
10	8	0.	3.
10	9	3.	1.
10	10	-1.	-4.

dgrf75

10 6371.2 1975.0

1	0	-30100.	0.
1	1	-2013.	5675.
2	0	-1902.	0.
2	1	3010.	-2067.
2	2	1632.	-68.
3	0	1276.	0.
3	1	-2144.	-333.
3	2	1260.	262.
3	3	830.	-223.
4	0	946.	0.
4	1	791.	191.
4	2	438.	-265.
4	3	-405.	39.
4	4	216.	-288.
5	0	-218.	0.
5	1	356.	31.
5	2	264.	148.
5	3	-59.	-152.
5	4	-159.	-83.
5	5	-49.	88.
6	0	45.	0.
6	1	66.	-13.
6	2	28.	99.
6	3	-198.	75.
6	4	1.	-41.
6	5	6.	-4.
6	6	-111.	11.
7	0	71.	0.
7	1	-56.	-77.
7	2	1.	-26.
7	3	16.	-5.
7	4	-14.	10.
7	5	0.	22.
7	6	12.	-23.
7	7	-5.	-12.
8	0	14.	0.
8	1	6.	6.
8	2	-1.	-16.
8	3	-12.	4.
8	4	-8.	-19.
8	5	4.	6.
8	6	0.	18.
8	7	10.	-10.
8	8	1.	-17.
9	0	7.	0.
9	1	10.	-21.
9	2	2.	16.
9	3	-12.	7.
9	4	10.	-4.
9	5	-1.	-5.
9	6	-1.	10.
9	7	4.	11.
9	8	1.	-3.
9	9	-2.	1.
10	0	-3.	0.
10	1	-3.	1.
10	2	2.	1.
10	3	-5.	3.
10	4	-2.	4.
10	5	5.	-4.
10	6	4.	-1.
10	7	1.	-1.
10	8	0.	3.
10	9	3.	1.
10	10	-1.	-5.

dgrf80

10 6371.2 1980.0

1	0	-29992.	0.
1	1	-1956.	5604.
2	0	-1997.	0.
2	1	3027.	-2129.
2	2	1663.	-200.
3	0	1281.	0.
3	1	-2180.	-336.
3	2	1251.	271.
3	3	833.	-252.
4	0	938.	0.
4	1	782.	212.
4	2	398.	-257.
4	3	-419.	53.
4	4	199.	-297.
5	0	-218.	0.
5	1	357.	46.
5	2	261.	150.
5	3	-74.	-151.
5	4	-162.	-78.
5	5	-48.	92.
6	0	48.	0.
6	1	66.	-15.
6	2	42.	93.
6	3	-192.	71.
6	4	4.	-43.
6	5	14.	-2.
6	6	-108.	17.
7	0	72.	0.
7	1	-59.	-82.
7	2	2.	-27.
7	3	21.	-5.
7	4	-12.	16.
7	5	1.	18.
7	6	11.	-23.
7	7	-2.	-10.
8	0	18.	0.
8	1	6.	7.
8	2	0.	-18.
8	3	-11.	4.
8	4	-7.	-22.
8	5	4.	9.
8	6	3.	16.
8	7	6.	-13.
8	8	-1.	-15.
9	0	5.	0.
9	1	10.	-21.
9	2	1.	16.
9	3	-12.	9.
9	4	9.	-5.
9	5	-3.	-6.
9	6	-1.	9.
9	7	7.	10.
9	8	2.	-6.
9	9	-5.	2.
10	0	-4.	0.
10	1	-4.	1.
10	2	2.	0.
10	3	-5.	3.
10	4	-2.	6.
10	5	5.	-4.
10	6	3.	0.
10	7	1.	-1.
10	8	2.	4.
10	9	3.	0.
10	10	0.	-6.

dgrf85

10 6371.2 1985.0

1	0	-29873.	0.
1	1	-1905.	5500.
2	0	-2072.	0.
2	1	3044.	-2197.
2	2	1687.	-306.
3	0	1296.	0.
3	1	-2208.	-310.
3	2	1247.	284.
3	3	829.	-297.
4	0	936.	0.
4	1	780.	232.
4	2	361.	-249.
4	3	-424.	69.
4	4	170.	-297.
5	0	-214.	0.
5	1	355.	47.
5	2	253.	150.
5	3	-93.	-154.
5	4	-164.	-75.
5	5	-46.	95.
6	0	53.	0.
6	1	65.	-16.
6	2	51.	88.
6	3	-185.	69.
6	4	4.	-48.
6	5	16.	-1.
6	6	-102.	21.
7	0	74.	0.
7	1	-62.	-83.
7	2	3.	-27.
7	3	24.	-2.
7	4	-6.	20.
7	5	4.	17.
7	6	10.	-23.
7	7	0.	-7.
8	0	21.	0.
8	1	6.	8.
8	2	0.	-19.
8	3	-11.	5.
8	4	-9.	-23.
8	5	4.	11.
8	6	4.	14.
8	7	4.	-15.
8	8	-4.	-11.
9	0	5.	0.
9	1	10.	-21.
9	2	1.	15.
9	3	-12.	9.
9	4	9.	-6.
9	5	-3.	-6.
9	6	-1.	9.
9	7	7.	9.
9	8	1.	-7.
9	9	-5.	2.
10	0	-4.	0.
10	1	-4.	1.
10	2	3.	0.
10	3	-5.	3.
10	4	-2.	6.
10	5	5.	-4.
10	6	3.	0.
10	7	1.	-1.
10	8	2.	4.
10	9	3.	0.
10	10	0.	-6.

dgrf90

```
10  6371.2 1990.0
  1  0 -29775.      0.
  1  1  -1848.   5406.
  2  0  -2131.      0.
  2  1   3059.  -2279.
  2  2   1686.   -373.
  3  0   1314.      0.
  3  1  -2239.   -284.
  3  2   1248.    293.
  3  3    802.   -352.
  4  0    939.      0.
  4  1    780.    247.
  4  2    325.   -240.
  4  3   -423.     84.
  4  4    141.   -299.
  5  0   -214.      0.
  5  1    353.     46.
  5  2    245.    154.
  5  3   -109.   -153.
  5  4   -165.    -69.
  5  5    -36.     97.
  6  0     61.      0.
  6  1     65.    -16.
  6  2     59.     82.
  6  3   -178.     69.
  6  4      3.    -52.
  6  5     18.      1.
  6  6    -96.     24.
  7  0     77.      0.
  7  1    -64.    -80.
  7  2      2.    -26.
  7  3     26.      0.
  7  4     -1.     21.
  7  5      5.     17.
  7  6      9.    -23.
  7  7      0.     -4.
  8  0     23.      0.
  8  1      5.     10.
  8  2     -1.    -19.
  8  3    -10.      6.
  8  4    -12.    -22.
  8  5      3.     12.
  8  6      4.     12.
  8  7      2.    -16.
  8  8     -6.    -10.
  9  0      4.      0.
  9  1      9.    -20.
  9  2      1.     15.
  9  3    -12.     11.
  9  4      9.     -7.
  9  5     -4.     -7.
  9  6     -2.      9.
  9  7      7.      8.
  9  8      1.     -7.
  9  9     -6.      2.
 10  0     -3.      0.
 10  1     -4.      2.
 10  2      2.      1.
 10  3     -5.      3.
 10  4     -2.      6.
 10  5      4.     -4.
 10  6      3.      0.
 10  7      1.     -2.
 10  8      3.      3.
 10  9      3.     -1.
 10 10      0.     -6.
```

igrf95

```
10  6371.2 1995.0
  1  0 -29682.      0.0
  1  1  -1789.   5318.
  2  0  -2197.      0.0
  2  1   3074.  -2356.
  2  2   1685.   -425.
  3  0   1329.      0.0
  3  1  -2268.   -263.
  3  2   1249.    302.
  3  3    769.   -406.
  4  0    941.      .0
  4  1    782.    262.
  4  2    291.   -232.
  4  3   -421.     98.
  4  4    116.   -301.
  5  0   -210.      .0
  5  1    352.     44.
  5  2    237.    157.
  5  3   -122.   -152.
  5  4   -167.    -64.
  5  5    -26.     99.
  6  0     66.      .0
  6  1     64.    -16.
  6  2     65.     77.
  6  3   -172.     67.
  6  4      2.    -57.
  6  5     17.      4.
  6  6    -94.     28.
  7  0     78.     -.0
  7  1    -67.    -77.
  7  2      1.    -25.
  7  3     29.      3.
  7  4      4.     22.
  7  5      8.     16.
  7  6     10.    -23.
  7  7     -2.     -3.
  8  0     24.      .0
  8  1      4.     12.
  8  2     -1.    -20.
  8  3     -9.      7.
  8  4    -14.    -21.
  8  5      4.     12.
  8  6      5.     10.
  8  7      0.    -17.
  8  8     -7.    -10.
  9  0      4.      .0
  9  1      9.    -19.
  9  2      1.     15.
  9  3    -12.     11.
  9  4      9.     -7.
  9  5     -4.     -7.
  9  6     -2.      9.
  9  7      7.      7.
  9  8      0.     -8.
  9  9     -6.      1.
 10  0     -3.      .0
 10  1     -4.      2.
 10  2      2.      1.
 10  3     -5.      3.
 10  4     -2.      6.
 10  5      4.     -4.
 10  6      3.      0.
 10  7      1.     -2.
 10  8      3.      3.
 10  9      3.     -1.
 10 10      0.     -6.
```

igrf95s

```
8  6371.2 2000.0
  1  0      17.6      0.
  1  1      13.0    -18.3
  2  0     -13.2      0.
  2  1       3.7    -15.0
  2  2      -0.8     -8.8
  3  0       1.5      0.
  3  1      -6.4      4.1
  3  2      -0.2      2.2
  3  3      -8.1    -12.1
  4  0       0.8      0.
  4  1       0.9      1.8
  4  2      -6.9      1.2
  4  3       0.5      2.7
  4  4      -4.6     -1.0
  5  0       0.8      0.
  5  1       0.1      0.2
  5  2      -1.5      1.2
  5  3      -2.0      0.3
  5  4      -0.1      1.8
  5  5       2.3      0.9
  6  0       0.5      0.
  6  1      -0.4      0.3
  6  2       0.6     -1.6
  6  3       1.9     -0.2
  6  4      -0.2     -0.9
  6  5      -0.2      1.0
  6  6        .0      2.2
  7  0      -0.2      0.
  7  1      -0.8      0.8
  7  2      -0.6      0.2
  7  3       0.6      0.6
  7  4       1.2     -0.4
  7  5       0.1      0.0
  7  6       0.2     -0.3
  7  7      -0.6      0.
  8  0       0.3      0.
  8  1      -0.2      0.4
  8  2       0.1     -0.2
  8  3       0.4      0.2
  8  4      -1.1      0.7
  8  5       0.3      0.0
  8  6       0.2     -1.2
  8  7      -0.9     -0.7
  8  8      -0.3     -0.6
```

Appendix B

Tilt Dependent External Magnetic Field Routines

These listings are the 1977 Olson-Pfitzer tilt dependent routine including the routine that combines the internal and the external field. These routines are included here for completeness. They were not developed or changed as a part of this effort. They are, however, necessary in order for the remaining software to function properly.

```
C      BMNIGRF -- A TEST ROUTINE TO CHECK THE OPERATION OF THE TILT
C      DEPENDENT MODEL AND IT COMBINATION WITH THE IGRF MAIN FIELD
C
       DIMENSION X(3),B(3)
       COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW

C
C      SET UP THE YEAR FOR THE MAIN FIELD ROUTINE
C
       YEAR=1985.
C
C      SET THE SWITCH TO USE EXTERNAL PLUS INTERNAL FIELD
C
       JSW=1
C
C      SET THE SWITCH TO USE INPUT AND OUTPUT IN CARTESIAN COORDS
C
       KODE=1
C
C      SET UP A CARTESIAN COORDINATE TEST LOOP
C
C      SET UP DATE AND TIME
       DO 200 ID=1,2
       DAYYR=90*ID
       DO 190 IUT=1,3
       UT=IUT*6-6
C
C      PRINT PAGE HEADER
       WRITE (6,110)
 110   FORMAT(77H1DAYOFYR   UT       X        Y        Z         BX         BY
     *         BZ         BMAG,/)
C      SET UP POSITION IN CARTESIAN COORDS
       DO 180 IZ=1,3
       X(3)=3*IZ-6
       DO 170 IY=1,3
       X(2)=3*IY-6
       DO 160 IX=1,6
       X(1)=4*IX-14
C
C      GET THE MAGNETIC FIELD VALUES
       CALL BMNEXT(X,B,BMAG)
       WRITE (6,120) DAYYR,UT,X,B,BMAG
 120   FORMAT(F6.0,4F8.2,4F10.5)
 160   CONTINUE
 170   CONTINUE
 180   CONTINUE
 190   CONTINUE
 200   CONTINUE
C
C      SET UP FOR SPHERICAL COORDINATES
C
1000       KODE=2
C
C      SET DATE AND TIME
       DO 300 ID=1,2
       DAYYR=90*ID
       DO 290 IUT=1,3
       UT=IUT*6-6
C
C      PRINT PAGE HEADER
```

```
      WRITE (6,210)
 210  FORMAT(77H1DAYOFYR    UT        R      THETA     PHI         BR        BTHE
     *TA      BPHI       BMAG,/)
C
C     SET UP POSITIONS IN SPHERICAL COORDS
      DO 280 IR=1,3
      X(1)=IR*3
      DO 270 IT=1,3
      X(2)=IT*45
      DO 260 IP=1,6
      X(3)=(IP-1)*60
C
C     GET THE MAGNETIC FIELD
      CALL BMNEXT(X,B,BMAG)
      WRITE(6,120) DAYYR,UT,X,B,BMAG
 260  CONTINUE
 270  CONTINUE
 280  CONTINUE
 290  CONTINUE
 300  CONTINUE
      END
```

```
      SUBROUTINE BMNEXT(XX,B,BMAG)
C
C     PURPOSE
C        TO DETERMINE THE MAIN MAGNETIC FIELD PLUS THE EXTERNAL
C        FIELD
C
C     METHOD
C        DETERMINES THE VECTOR MAGNETIC FIELD IN GEOGRAPHIC
C        COORDINATES USING A SPHERICAL COORDINATE EXPANSION OF THE
C        EARTHS INTERNAL FIELD AND A CARTESIAN COORDINATE EXPANSION
C        OF THE BOUNDARY, TAIL AND RING CURRENT FIELDS IN SOLAR
C        MAGNETIC COORDINATES
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY CONTAINING THE POSITION IN GEOGRAPHIC
C                COORDINATES
C                IF KODE = 1
C                XX(1)=X, XX(2)=Y, XX(3)=Z, WHERE X, Y, Z ARE IN EARTH
C                RADII.  THE DIRECTION OF Z IS ALONG THE EARTHS ROTATION
C                AXIS TOWARDS THE GEOGRAPHIC NORTH POLE.  THE DIRECTION
C                OF X IS TO THE GREENWHICH MERIDIAN IN THE EQUATORIAL
C                PLANE.  THE Y AXIS IS IN THE EQUATORIAL PLANE NORMAL
C                TO X AND Z IN A RIGHT HANDED SENSE.
C                IF KODE = 2
C                XX(1)=R, GEOCENTRIC RADIUS IN EARTH RADII,
C                XX(2)=THETAG, COLATITUDE IN DEGREES,
C                XX(3)=PHIG, LONGITUDE IN DEGREES
C
C     INPUT -- COMMON BLOCK BXYZCM
C        UT      THE CURRENT UNIVERSAL TIME IN HOURS
C        KODE    A FLOW CONTROL VARIABLE.  KODE EQUAL TO ONE MEANS THAT
C                INPUT AND OUTPUT ARE IN CARTESIAN COORDINATES.  KODE
C                EQUAL TO TWO MEANS THAT INPUT AND OUTPUT ARE SPHERICAL
C                COORDINATES.
C        DAYYR   THE NUMBER OF THE DAY OF YEAR
C        JSW     A FLOW CONTROL VARIABLE. IF JSW IS LESS THAN ZERO, THE
C                THE FIELD IS COMPUTED USING THE INTERNAL FIELD ONLY.
C                IF JSW IS GREATER THAN OR EQUAL TO ZERO THE FIELD
C                WILL BE COMPUTED USING THE INTERNAL PLUS EXTERNAL
C                FIELD.
C        YEAR    THE YEAR USED BY THE INTERNAL MAGNETIC FIELD ROUTINE
C                TO TAKE INTO ACCOUNT THE SECULAR VARIATIONS
C                (E.G. JULY 15, 1964 = 1964.54)
C                NOTE**** YEAR SHOULD BE CHANGED ONLY EVERY FEW DAYS OR
C                MONTHS.  NEW FIELD COEFFICIENTS MUST BE COMPUTED FOR
C                EVERY CHANGE IN YEAR.  THIS COULD CAUSE A LARGE INCREASE
C                IN COMPUTER TIME.  THE EARTHS FIELD CHANGES ONLY ABOUT
C                .001 GAUSS/YEAR AT THE EARTHS SURFACE.
C
C     OUTPUT -- ARGUMENT LIST
C        B       A REAL ARRAY CONTAINING THE COMPONENTS OF THE MAGNETIC
C                FIELD IN GAUSS AT THE CURRENT POSITION AND TIME
C                IF KODE = 1
C                B(1)=BX, B(2)=BY, B(3)=BZ  THE CARTESIAN COMPONENTS
C                OF THE MAGNETIC FIELD IN GEOGRAPHIC COORDINATES
C                IF KODE = 2
C                B(1)=BR, RADIAL COMPONENT OF THE FIELD, POSITIVE IN THE
C                DIRECTION OF INCREASING RADIUS.
C                B(2)=BTHETA, COMPONENT IN LATITUDE, POSITIVE IN THE
C                DIRECTION OF INCREASING COLATITUDE
```

```
C              B(3)=BPHI, COMPONENT IN LONGITUDE, POSITIVE IN THE
C              DIRECTION OF INCREASING LONGITUDE.
C        BMAG  THE MAGNITUDE OF THE MAGNETIC FIELD VECTOR IN UNITS OF
C              GAUSS.
C
C     OUTPUT -- COMMON BLOCK BXYZCM
C        XMLAT THE MAGNETIC LATITUDE AT THE CURRENT POSITION IN RADIANS
C
C     SUBROUTINE CONSTANTS
C        PICON THE NUMBER OF DEGREES PER RADIAN
C        SIN D THE SINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        COS D THE COSINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        C69   COSINE OF 69
C        S69   SINE OF 69
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE BXYZMU
C        SUBROUTINE ANGLE
C        SUBROUTINE SPIGRF
C
C     VARIABLES
C        AOR   INVERSE OF RADIUS VECTOR (AOR=1./R)
C        BGMX,BGMY,BGMZ  INTERMEDIATE VALUES OF THE MAGNETIC FIELD
C              VECTOR DURING COORDINATE TRANSFORMATION
C        BMX,BMY,BMZ  EXTERNAL MAGNETIC FIELD IN GEOMAGNETIC COORDINATES
C        BP,BR,BT  COMPONENTS OF INTERNAL FIELD IN SPHERICAL COORDINATES
C              BP IS LONGITUDINAL COMPONENT
C              BR IS RADIAL COMPONENT
C              BT IS LATITUDINAL COMPONENT
C        BX,BY,BZ  CARTESIAN COMPONENT OF EXTERNAL FIELD IN GEOGRAPHIC
C              COORDINATES
C        CP    COSINE OF COLATITUDE
C        CPS   COSINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C              GEOMAGNETIC COORDINATES
C        CT    COSINE OF GEOGRAPHIC LONGITUDE
C        DAYLST LAST DAY FOR WHICH TILT AND HOUR ANGLE WERE UPDATED
C        NMAX  MAXIMUM NUMBER OF TERMS USED BY INTERNAL FIELD ROUTINE
C              SET UP BY INTERNAL FIELD ROUTINE
C        PHIG  GEOGRAPHIC COLATITUDE
C        R     RADIUS VECTOR TO POSITION POINT
C        R2    R**2
C        SP    SINE OF COLATITUDE
C        SPS   SINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C              GEOMAGNETIC COORDINATES
C        ST    SINE OF LONGITUDE
C        THETAG GEOGRAPHIC LONGITUDE
C        TILT  TILT OF THE DIPOLE AXIS
C        UTLST LAST UNIVERSAL TIME FOR WHICH TILT AND HOUR ANGLE WERE
C              UPDATED
C        X     A REAL ARRAY HOLDING THE POSITION VECTOR IN SOLAR
C              MAGNETIC COORDINATES
C        XP,YP,ZP  POSITION VECTOR IN GEOMAGNETIC COORDINTES
C        XPP,YPP INTERMEDIATE POSITION COMPONENT DURING COORDINATE
C              TRANSFORMATION
C        YEARI TRANSMITS THE YEAR TO THE INTERNAL FIELD ROUTINE
C
C     VERSION 10/25/77
```

```
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER AT MCDONNELL
C     DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON BEACH CALIF.
C     PHONE (714) 896-3231.
C
      DIMENSION X(3),B(3),XX(3)
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      COMMON /GCOM/ ST,CT,SP,CP,AOR,BT,BP,BR,NMAX,YEARI
      DATA PICON/57.29577951/,SIND,COSD/.2027872954,.9792228106/,
     *S69,C69/.9335804265,.3583679495/,UTLST,DAYLST/2*123456./
C
C     UPDATE THE ROTATION HOUR ANGLE AND TILT ANGLE IF THE UNIVERSAL
C     TIME OR DAY OF YEAR HAS CHANGED SINCE THE LAST CALL
C
      IF(UT.EQ.UTLST.AND.DAYYR.EQ.DAYLST) GO TO 1
      UTLST=UT
      DAYLST=DAYYR
      CALL ANGLE (TILT,SPS,CPS)
 1    IF(KODE.GT.1) GO TO 3
C
C     DETERMINE THE SPHERICAL COORDINATES OF POSITION IF CARTESIAN
C     COORDINATES WERE ENTERED
C
      X(1)=XX(1)
      X(2)=XX(2)
      X(3)=XX(3)
      R2=X(1)**2+X(2)**2
      R=SQRT(X(3)**2+R2)
      R2=SQRT(R2)
      CT=X(3)/R
      ST=R2/R
      CP=X(1)/R2
      SP=X(2)/R2
      GO TO 5
C
C     DETERMINE THE CARTESIAN COORDINATES OF POSITION IF SPHERICAL
C     COORDINATES WERE ENTERED
C
 3    R=XX(1)
      THETAG=XX(2)/PICON
      PHIG=XX(3)/PICON
      CT=COS(THETAG)
      ST=SIN(THETAG)
      CP=COS(PHIG)
      SP=SIN(PHIG)
      X(1)=R*ST*CP
      X(2)=R*ST*SP
      X(3)=R*CT
 5    BX=0.
      BY=0.
      BZ=0.
C
C     IF THE EXTERNAL MAGNETIC FIELD IS TO BE USED IN THE COMPUTATION,
C     COMPUTE THE SOLAR MAGNETIC COORDINATES
C
      IF(JSW.LT.0) GO TO 9
C
C     FIRST ROTATION IS ABOUT THE Z-AXIS TROUGH AN ANGLE OF 291 DEGREES
C     (THE LONGITUDE OF THE MAGNETIC NORTH POLE)
C
      XPP=X(1)*C69-X(2)*S69
```

```
      YPP= X(1)*S69+X(2)*C69
C
C     SECOND ROTATION IS ABOUT THE NEW Y-AXIS THROUGH AN ANGLE OF 11.7
C     DEGREES (THE COLATITUDE OF THE MAGNETIC NORTH POLE)
C
      ZP=XPP*SIND+X(3)*COSD
      XP=XPP*COSD-X(3)*SIND
      YP=YPP
C
C     ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR ANGLE OF
C     THE SUN FROM THE PRIME MAGNETIC MERIDIAN (NEGATIVE ROTATION)
C
      X(1)=XP*CPS-YP*SPS
      X(2)=XP*SPS+YP*CPS
      X(3)=ZP
C
C     DETERMINE THE EXTERNAL MAGNETIC FIELD USING A TILT DEPENDENT
C     MAGNETIC FIELD
C
      CALL BXYZMU(X,B,TILT)
C
C     THE CARTESIAN COMPONENTS OF THE FIELD ARE IN SOLAR MAGNETIC
C     COORDINATES. THE COMPONENTS ARE NEEDED IN THE GEOGRAPHIC
C     COORDINATE SYSTEM
C
C     FIRST ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR
C     ANGLE OF THE SUN TO THE PRIME MAGNETIC MERIDIAN
C     (POSITIVE ROTATION) PUTS RESULTS INTO GEOMAGNETIC COORDINATES
C
      BMX=B(1)*CPS+B(2)*SPS
      BMY=-B(1)*SPS+B(2)*CPS
      BMZ=B(3)
C
C     SECOND ROTATION IS ABOUT THE MAGNETIC Y-AXIS THOUGH -11.7 DEGREES
C     COLATITUDE
C
      BGMX=BMX*COSD+BMZ*SIND
      BGMY=BMY
      BGMZ=-BMX*SIND+BMZ*COSD
C
C     THIRD ROTATION IS ABOUT THE NEW Z-AXIS THROUGH -291 DEGREES
C
      BX=BGMX*C69+BGMY*S69
      BY=-BGMX*S69+BGMY*C69
      BZ=BGMZ
C
C     DETERMINE THE MAIN FIELD
C
 9    CONTINUE
      AOR=1./R
      YEARI=YEAR
      CALL SPIGRF
      IF(KODE.GT.1) GO TO 10
C
C     IF THE OUTPUT IS TO BE IN CARTESIAN GEOGRAPHIC COORDINATES CONVERT
C     THE MAIN MAGNETIC FIELD AND ADD
C
      B(1)=(BX+CP*(ST*BR+CT*BT)-SP*BP)*0.00001
      B(2)=(BY+SP*(ST*BR+CT*BT)+CP*BP)*0.00001
      B(3)=(BZ+CT*BR-ST*BT)*0.00001
```

```
      GO TO 20
C
C     IF OUTPUT IS TO BE IN SPHERICAL GEOGRAPHIC CONVERT THE EXTERNAL
C     FIELD AND ADD
C
 10   B(1)=(BR+(BX*CP+BY*SP)*ST+BZ*CT)*0.00001
      B(2)=(BT+(BX*CP+BY*SP)*CT-BZ*ST)*0.00001
      B(3)=(BP+BY*CP-BX*SP)*0.00001
C
C     DETERMINE THE MAGNITUDE OF THE FIELD VECTOR
C
 20   BMAG=SQRT(B(1)**2+B(2)**2+B(3)**2)
      RETURN
      END
```

```
      SUBROUTINE ANGLE(TILT,SINPHE,COSPHE)
C
C     PURPOSE
C        THIS ROUTINE CALCULATES THE ANGLE BETWEEN THE MAGNETIC DIPOLE
C        AXIS AND THE SUN-EARTH LINE AS WELL AS THE ROTATION SINES
C        AND COSINES TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C        COORDINATES
C
C     METHOD
C        MAGNETIC COORDINATES HAVE THEIR ORIGIN AT THE CENTER OF THE
C        EARTH WITH THE Z AXIS ALLIGNED THROUGH THE GEOMAGNETIC NORTH
C        POLE.  IN GEOMAGNETIC COORDINATES THE X AXIS IS IN THE
C        PLANE PASSING THROUGH THE DIPOLE AXIS AND THE GEOGRAPHIC
C        AXIS (ABOUT 69 DEGREES WEST LONG.).  IN SOLAR MAGNETIC
C        COORDINATES X AXIS LIES IN THE PLANE CONTAINING THE SUN
C        EARTH LINE AND THE Z AXIS (POSITIVE X AXIS HAS A LARGE
C        COMPONENT IN THE SOLAR DIRECTION). THE Y AXIS IS ORTHOGONAL
C        TO THE X AND Z AXIS SUCH THAT X, Y AND Z FORM A RIGHT
C        HANDED SYSTEM.  THE ECCLIPTIC COORDINATE SYSTEM HAS ITS
C        Z AXIS ALONG THE ECCLIPTIC NORTH POLE (THROUGH THE CENTER
C        OF THE EARTH AND PERPENDICULAR TO THE EARTHS ORBITAL PLANE)
C        THE X AXIS POINTS TOWARD THE SUN AND Y FORMS A RIGHT HANDED
C        COORDINATE SYSTEM.  IN THIS ROUTINE IN ORDER TO REDUCE
C        COMPUTER TIME THE APPROXIMATION OF A CIRCULAR EARTH ORBIT
C        AROUND THE SUN IS MADE.
C
C     INPUT -- COMMON BLOCK BXYZCM
C        DAYYR  IS THE DAY OF YEAR (1.-366.).  IT MUST BE A WHOLE
C               NUMBER.  DAY 1 IS JANUARY 1.
C        UT     THE UNIVERSAL TIME IN HOURS (0.0000-24.00000)
C
C     OUTPUT -- PARAMETER LIST
C        TILT   THE TILT OF THE DIPOLE AXIS IN DEGREES.
C               TILT = 90. - PSI, WHERE PSI IS THE ANGLE BETWEEN
C               THE MAGNETIC DIPOLE AXIS AND THE SOLAR DIRECTION.
C        SINPHE THE SINE OF THE ROTATION ANGLE ABOUT THE MAGNETIC
C               Z AXIS TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C               COORDINATES
C        COSPHE THE COSINE OF THE ROTATION ANGLE ABOUT THE MAGNETIC
C               Z AXIS TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C               COORDINATES
C
C     CONSTANTS
C        PI2    PI / 2.
C        CON    180. / PI CONVERTS RADIANS TO DEGREES
C        SALF   SINE (11.7)  INCLINATION OF MAGNETIC Z TO GEOGRAPHIC Z
C        CALF   COSINE (11.7)
C        SGAM   SIN (23.5) INCLINATION OF ROTATION AXIS TO ECLIPTIC Z
C        CGAM   COSINE (23.5)
C        SASG   SALF * SGAM
C        SACG   SALF * CGAM
C        CASG   CALF * SGAM
C        CACG   CALF * CGAM
C        W      EARTHS ANGULAR ROTATION FREQUENCY CORRECTED FOR ITS
C               ONCE A YEAR ROTATION ABOUT THE SUN (UNITS ARE 1/HOURS)
C
C     CALLING SUBROUTINES
C        SUBROUTINE BMNEXT
C
C     VARIABLES
```

```
C        WT      INSTANTANEOUS ROTATION ANGLE AT THE SPECIFIED UNIVERSAL
C                TIME AND DAY OF YEAR
C        CWT     WT/365.256
C        XX,XY,XZ COMPONENTS OF THE GEOMAGNEITIC X AXIS IN ECCLIPTIC
C                COORDINATES
C        ZX,ZY,ZZ COMPONENTS OF THE DIPOLE AXIS IN ECCLIPTIC COORDINATES
C        OSP,SSMLT,CSMLT,SBWT,CBWT,SMLSWT,SMLCWT,CMLSWT,CMLCWT ARE
C                SINES AND COSINES AND THEIR PRODUCTS AND ARE SET UP
C                TO MINIMIZE COMPUTER TIME
C
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      DATA  IFIRST/0/
C
C     THE FIRST TIME TROUGH THE SUBROUTINE SET UP THE FIXED CONSTANTS
      IF(IFIRST.NE.0) GO TO 10
      IFIRST=1
      PI2=ATAN2(0.,-1.)/2.
      CON=90./PI2
      SALF=SIN(11.7/CON)
      CALF=COS(11.7/CON)
      SGAM=SIN(23.5/CON)
      CGAM=COS(23.5/CON)
      SASG=SALF*SGAM
      SACG=SALF*CGAM
      CASG=CALF*SGAM
      CACG=CALF*CGAM
      W=PI2/6.*(1.+1./365.256)
C
C     MAIN ENTRY POINT.  SET UP THE THE SINES AND COSINES REQUIRED
C     BY THE TRANSFORMATIONS.
 10   WT=(UT-16.6+(DAYYR-172.)*24.)*W
      CWT=-WT/365.256
      SSMLT=SIN(WT)
      CSMLT=COS(WT)
      SBWT=SIN(CWT)
      CBWT=COS(CWT)
      SMLSWT=SSMLT*SBWT
      SMLCWT=SSMLT*CBWT
      CMLSWT=CSMLT*SBWT
      CMLCWT=CSMLT*CBWT
C
C     DETERMINE THE COMPONENTS OF THE DIPOLE AXIS IN ECCLIPTIC
C     COORDINATES
      ZX=CASG*CBWT+SACG*CMLCWT-SALF*SMLSWT
      ZY=CASG*SBWT+SACG*CMLSWT+SALF*SMLCWT
      ZZ=CACG-SASG*CSMLT
C
C     CALCULATE THE TILT ANGLE
      PSI=ACOS(ZX)
      OSP=1./(SIN(PSI))
      TILT=CON*(PI2-PSI)
C
C     DETEMINE THE COMPONENTS OF THE GEOMAGNETIC X AXIS IN ECCLIPTIC
C     COORDINATES
      XX=CACG*CMLCWT-SASG*CBWT-CALF*SMLSWT
      XY=CACG*CMLSWT-SASG*SBWT+CALF*SMLCWT
      XZ=-CASG*CSMLT-SACG
C
C     OBTAIN THE ROTATION SINES AND COSINES
      SINPHE=(XY*ZZ-XZ*ZY)*OSP
```

```
COSPHE=(XX*(ZZ*ZZ+ZY*ZY)-ZX*(XY*ZY+XZ*ZZ))*OSP
RETURN
END
```

```
      SUBROUTINE BXYZMU(XX,BF,TILT)
C
C  VERSION 11/01/76
C
C     PURPOSE
C       TO CALCULATE THE CONTRIBUTION TO THE EARTHS MAGNETIC FIELD BY
C       SOURCES EXTERNAL TO THE EARTH.  NO INTERNAL FIELD IS INCLUDED
C       IN THIS ROUTINE.
C
C     METHOD
C        THE ROUTINE INCLUDES THE FIELD CONTRIBUTIONS FROM THE
C        MAGNETOPAUSE CURRENTS, AND CURRENTS DISTRIBUTED THROUGHOUT
C        THE MAGNETOSPHERE (THE TAIL AND RING CURRENTS).  IT IS VALID
C        FOR ALL TILTS OF THE EARTHS DIPOLE AXIS AND IS VALID DURING
C        QUIET MAGNETIC CONDITIONS.
C        A GENERALIZED ORTHONORMAL LEAST SQUARES PROGRAM WAS USED
C        TO FIT THE COEFFICIENTS OF A POWER SERIES (INCLUDING
C        EXPONENTIAL TERMS) THROUGH FOURTH ORDER IN SPACE AND
C        THIRD ORDER IN TILT.  THIS EXPANSION HAS BEEN OPTIMIZED
C        FOR THE NEAR EARTH REGION AND IS VALID TO 15 EARTH RADII.
C        OUTSIDE OF THIS REGION THE FIELD DIVERGES RAPIDLY AND A
C        TEMPLATE SETS THE FIELD TO ZERO.  IN ORDER TO IMPROVE
C        COMPUTATIONAL SPEED THE FIELD IS SET TO ZERO BELOW 2 EARTH
C        RADII.  (IN THIS REGION THE EARTHS INTERNAL FIELD DOMINATES
C        AND THE VARIATIONS EXCPRESSED BY THIS EXPANSION IS NOT
C        SUFFICIENTLY ACCURATE THE PREDICT VARIATIONS ON THE EARTHS
C        SURFACE)
C
C        THE POWER SERIES REPRESENTING THE MAGNETIC FIELD IS
C        BX=SUM OVER I,J,K OF ( A(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)
C               + B(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5,J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J + K IS LESS THAN OR EQUAL TO 9
C        BY=SUM OVER I,J,K OF ( C(I,J,K)*X**(I-1)*Y**(2*J-1)*Z**(K-1)
C               + D(I,J,K)*X**(I-1)*Y**(2*J-1)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5, J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J+1 + K IS LESS THAN OR EQUAL TO 9
C        BZ=SUM OVER I,J,K OF ( E(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)
C               + F(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5, J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J + K IS LESS THAN OR EQUAL TO 9
C        THE COEFFICIENTS A-F ARE DEPENDENT ONLY ON POSITION AND
C        ARE RECALCULATED EACH TIME THE TILT OF THE DIPOLE IS CHANGED.
C        THE COEEFICIENTS A-F ARE DETERMINED FROM THE TILT DEPENDENT
C        CONSTANTS AA-FF BY THE FOLLOWING EXPRESSIONS
C        A(I,J,K)=AA(I,J,K,1)*TILT**(K-1-(K-1)/2*2)
C                 +AA(I,J,K,2)*TILT**(K+1-(K-1)/2*2)
C        B(I,J,K)=BB......
C        C(I,J,K)=CC......
C        D(I,J,K)=DD......
C        E(I,J,K)=EE(I,J,K,1)*TILT**(K-(K)/2*2)
C                 +EE(I,J,K,2)*TILT**(K+2-(K)/2*2)
C        F(I,J,K)=FF......
C
C     INPUT -- CALLING SEQUENCE
C        XX      A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C                FIELD IS TO BE EVALUATED.  XX(1), XX(2), XX(3) ARE
C                RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC
C                COORDINATES IN EARTH RADII. Z IS ALONG THE EARTHS
C                NORTH DIPOLE AXIS, X IS PERPENDICULAR TO Z AND IN THE
```

```
C              PLANE CONTAINING THE Z AXIS AND THE SUN-EARTH LINE,
C              Y IS PERPENDICULAR TO X AND Z FORMING A RIGHT HANDED
C              COORDINATE SYSTEM. X IS POSITIVE IN THE SOLAR DIRECTION.
C        TILT  IS THE TILT OF THE DIPOLE AXIS IN DEGREES.  IT IS
C              THE COMPLEMENT OF THE ANGLE BETWEEEN THE NORTH DIPOLE
C              AXIS AND THE SOLAR DIRECTION (PSI). TILT=90-PSI.
C
C     OUTPUT -- CALLING SEQUENCE
C        BF    A REAL ARRAY CONTAINING THE X, Y, AND Z COMPONENTS OF
C              THE MAGNETOSPHERIC MAGNETIC FIELD IN GAMMA. BF(1),
C              BF(2) AND BF(3) ARE THE BX, BY, BZ COMPONENTS.
C
C     CONSTANTS
C        AA,BB,CC,DD,EE,FF ARE REAL ARRAYS CONTAING THE TILT DEPENDED
C              COEFFICIENTS.
C              AA(I,J,K,L) ARE STORED SUCH THAT L VARIES MOST RAPIDLY
C              FOLLOWED IN ORDER BY K, J AND I.  I VARIES THE SLOWEST.
C              THE ARRAY IS CLOSE PACKED AND ALL COEFFICIENTS THAT
C              ARE ZERO BECAUSE OF SYMMETRY OR BECAUSE THE CROSS TERM
C              POWER IS TOO LARGE ARE DELETED.
C
C     VARIABLES
C        A,B,C,D,E,F  THE TILT INDEPENDENT COEFFICIENTS.  THEIR USE
C              IS DESCRIBED UNDER METHOD.
C        ITA   A REAL ARRAY WHICH CONTAINS THE SYMMETRY OF THE TILT
C              DEPENDENCE FOR EACH OF THE A AND B COEFFICIENTS
C              ITA(1) HAS THE SYMMETRY INFORMATION FOR A(1,1,1,1)
C                     AND A(1,1,1,2)
C              ITA(2) HAS THE SYMMETRY INFORMATION FOR A(1,1,2,1)
C                     AND A(1,1,2,2)  ETC.
C              IF ITA = 1 TILT SYMMETRY IS EVEN WITH RESPECT TO Z SYM.
C              IF ITA = 2 TILT SYMMETRY IS ODD WITH RESPECT TO Z SYM.
C        ITB   SYMMETRY POINTER FOR C AND D ARRAYS
C        ITC   SYMMETRY POINTER FOR E AND F ARRAYS
C        X     X COMPONENT OF POSITION
C        Y     Y COMPONENT OF POSITION
C        Z     Z COMPONENT OF POSITION
C        Y2    Y**2
C        Z2    Z**2
C        R2    X**2 + Y**2 + Z**2
C        R     SQRT(R2)
C        I     DO LOOP VARIABLE.  IN THE FIELD EXPANSION LOOP IT
C              REPRESENTS THE POWER TO WHICH X IS CURRENTLY RAISED
C              I.E. X**(I-1)
C        J     DO LOOP VARIABLE.  ALSO Y**(2*J-2)
C        K     DO LOOP VARIABLE.  ALSO Z**(K-1)
C        XB    X**(I-1)
C        YEXB  X**(I-1)*Y**(2*J-2)
C        ZEYEXB X**(I-1)*Y**(2*J-2)*Z**(K-1)
C        IJK   I + 2*J + K
C        II    POINTS TO THE ARRAY LOCATION WHERE THE CURRENT POWER
C              SERIES COEFFICIENT FOR BX IS LOCATED
C        JJ    BY COEFFICIENT LOCATION POINTER
C        KK    BZ COEFFICIENT LOCATION POINTER
C        BX,BY,BZ ARE USED TO CONSTRUCT THE MAGNETIC FIELD WITHIN THE
C              POWER SERIES LOOP.
C        EXPR  EXP(-.06*R2)
C        TILTL HOLDS THE LAST VALUE OF THE TILT FOR WHICH THE TILT
C              INDEPENDENT COEFFICIENTS A-F WERE CALCULATED
C        TT    A REAL ARRAY HOLDING THE POWERS OF THE TILT.
```

```
C               TT(1)=TILT**0, TT(2)=TILT**1, ETC.
C        CON    =0 FOR R LESS THAN 2
C               =1 FOR R GREATER THAN 2.5
C               GOES FROM 0 TO 1 IN THE REGION 2 TO 2.5
C
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER OR W. P. OLSON
C     AT MCDONNEL DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON
C     CALIF., PHONE (714) 896-3231.
C
      DIMENSION BF(3),XX(3),AA(64),BB(64),CC(44),DD(44),EE(64),FF(64),
     *A(32),B(32),C(22),D(22),E(32),F(32),TT(4),ITA(32),ITB(22),ITC(32)
      DATA (ITA(I),I=1,32) /2,1,2,1,2,2,1,2,1,2,1,2,1,2,1,2,2,1,2,2,2,1,
     *2,1,2,1,2,1,2,2,2,1/
      DATA (ITB(I),I=1,22) /2,1,2,1,2,2,1,2,2,2,1,2,1,2,1,2,1,2,2,2,1,2/
      DATA (ITC(I),I=1,32) /1,2,1,2,1,1,2,1,2,1,2,1,2,1,2,1,1,2,1,1,1,2,
     *1,2,1,2,1,2,1,1,1,2/
      DATA (AA(I),I=1,64)/-2.26836E-02,-1.01863E-04, 3.42986E+00,       TOTAL
     *-3.12195E-04, 9.50629E-03,-2.91512E-06,-1.57317E-03, 8.62856E-08, TOTAL
     *-4.26478E-05, 1.62924E-08,-1.27549E-04, 1.90732E-06,-1.65983E-02, TOTAL
     * 8.46680E-09,-5.55850E-05, 1.37404E-08, 9.91815E-05, 1.59296E-08, TOTAL
     * 4.52864E-07,-7.17669E-09, 4.98627E-05, 3.33662E-10,-5.97620E-02, TOTAL
     * 1.60669E-05,-2.29457E-01,-1.43777E-04, 1.09403E-03,-9.15606E-07, TOTAL
     * 1.60658E-03,-4.01198E-07,-3.15064E-06, 2.03125E-09, 4.92887E-04, TOTAL
     *-1.80676E-07,-1.12022E-03, 5.98568E-07,-5.90009E-06, 5.16504E-09, TOTAL
     *-1.48737E-06, 4.83477E-10,-7.44379E-04, 3.82472E-06, 7.41737E-04, TOTAL
     *-1.31468E-05,-1.24729E-04, 1.92930E-08,-1.91764E-04,-5.30371E-08, TOTAL
     * 1.38186E-05,-2.81594E-08, 7.46386E-06, 2.64404E-08, 2.45049E-04, TOTAL
     *-1.81802E-07,-1.00278E-03, 1.98742E-06,-1.16425E-05, 1.17556E-08, TOTAL
     *-2.46079E-06,-3.45831E-10, 1.02440E-05,-1.90716E-08,-4.00855E-05, TOTAL
     * 1.25818E-07/
      DATA (BB(I),I=1,64)/ 9.47753E-02, 1.45981E-04,-1.82933E+00,       TOTAL
     * 5.54882E-04, 5.03665E-03,-2.07698E-06, 1.10959E-01,-3.45837E-05, TOTAL
     *-4.40075E-05, 5.06464E-07,-1.20112E-03, 3.64911E-06, 1.49849E-01, TOTAL
     *-7.44929E-05, 2.46382E-04, 9.65870E-07,-9.54881E-04, 2.43647E-07, TOTAL
     * 3.06520E-04, 3.07836E-07, 6.48301E-03, 1.26251E-06,-7.09548E-03, TOTAL
     *-1.55596E-05, 3.06465E+00,-7.84893E-05, 4.95145E-03, 3.71921E-06, TOTAL
     *-1.52002E-01, 6.81988E-06,-8.55686E-05,-9.01230E-08,-3.71458E-04, TOTAL
     * 1.30476E-07,-1.82971E-01, 1.51390E-05,-1.45912E-04,-2.22778E-07, TOTAL
     * 6.49278E-05,-3.72758E-08,-1.59932E-03, 8.04921E-06, 5.38012E-01, TOTAL
     *-1.43182E-04, 1.50000E-04, 5.88020E-07,-1.59000E-02, 1.60744E-06, TOTAL
     * 3.17837E-04, 1.78959E-07,-8.93794E-03, 6.37549E-06, 1.27887E-03, TOTAL
     *-2.45878E-07,-1.93210E-01, 6.91233E-06,-2.80637E-04,-2.57073E-07, TOTAL
     * 5.78343E-05, 4.52128E-10, 1.89621E-04,-4.84911E-08,-1.50058E-02, TOTAL
     * 6.21772E-06/
      DATA (CC(I),I=1,44)/-1.88177E-02,-1.92493E-06,-2.89064E-01,       TOTAL
     *-8.49439E-05,-4.76380E-04,-4.52998E-08, 1.61086E-03, 3.18728E-07, TOTAL
     * 1.29159E-06, 5.52259E-10, 3.95543E-05, 5.61209E-08, 1.38287E-03, TOTAL
     * 5.74237E-07, 1.86489E-06, 7.10175E-10, 1.45243E-07,-2.97591E-10, TOTAL
     *-2.43029E-03,-6.70000E-07,-2.30624E-02,-6.22193E-06,-2.40815E-05, TOTAL
     * 2.01689E-08, 1.76721E-04, 3.78689E-08, 9.88496E-06, 7.33820E-09, TOTAL
     * 7.32126E-05, 8.43986E-08, 8.82449E-06,-6.11708E-08, 1.78881E-04, TOTAL
     * 8.62589E-07, 3.43724E-06, 2.53783E-09,-2.04239E-07, 8.16641E-10, TOTAL
     * 1.68075E-05, 7.62815E-09, 2.26026E-04, 3.66341E-08, 3.44637E-07, TOTAL
     * 2.25531E-10/
      DATA (DD(I),I=1,44)/ 2.50143E-03, 1.01200E-06, 3.23821E+00,       TOTAL
     * 1.08589E-05,-3.39199E-05,-5.27052E-07,-9.46161E-02,-1.95413E-09, TOTAL
     *-4.23614E-06, 1.43153E-08,-2.62948E-04, 1.05138E-07,-2.15784E-01, TOTAL
     *-2.20717E-07,-2.65687E-05, 1.26370E-08, 5.88917E-07,-1.13658E-08, TOTAL
     * 1.64385E-03, 1.44263E-06,-1.66045E-01,-1.46096E-05, 1.22811E-04, TOTAL
     * 3.43922E-08, 9.66760E-05,-6.32150E-07,-4.97400E-05,-2.78578E-08, TOTAL
```

```
     * 1.77366E-02, 2.05401E-07,-1.91756E-03,-9.49392E-07,-1.99488E-01, TOTAL
     *-2.07170E-06,-5.40443E-05, 1.59289E-08, 7.30914E-05, 3.38786E-08, TOTAL
     *-1.59537E-04,-1.65504E-07, 1.90940E-02, 2.03238E-06, 1.01148E-04, TOTAL
     * 5.20815E-08/
      DATA (EE(I),I=1,64)/-2.77924E+01,-1.01457E-03, 9.21436E-02,       TOTAL
     *-8.52177E-06, 5.19106E-01, 8.28881E-05,-5.59651E-04, 1.16736E-07, TOTAL
     *-2.11206E-03,-5.35469E-07, 4.41990E-01,-1.33679E-05,-7.18642E-04, TOTAL
     * 6.17358E-08,-3.51990E-03,-5.29070E-07, 1.88443E-06,-6.60696E-10, TOTAL
     *-1.34708E-03, 1.02160E-07, 1.58219E-06, 2.05040E-10, 1.18039E+00, TOTAL
     * 1.58903E-04, 1.86944E-02,-4.46477E-06, 5.49869E-02, 4.94690E-06, TOTAL
     *-1.18335E-04, 6.95684E-09,-2.73839E-04,-9.17883E-08, 2.79126E-02, TOTAL
     *-1.02567E-05,-1.25427E-04, 3.07143E-08,-5.31826E-04,-2.98476E-08, TOTAL
     *-4.89899E-05, 4.91480E-08, 3.85563E-01, 4.16966E-05, 6.74744E-04, TOTAL
     *-2.08736E-07,-3.42654E-03,-3.13957E-06,-6.31361E-06,-2.92981E-09, TOTAL
     *-2.63883E-03,-1.32235E-07,-6.19406E-06, 3.54334E-09, 6.65986E-03, TOTAL
     *-5.81949E-06,-1.88809E-04, 3.62055E-08,-4.64380E-04,-2.21159E-07, TOTAL
     *-1.77496E-04, 4.95560E-08,-3.18867E-04,-3.17697E-07,-1.05815E-05, TOTAL
     * 2.22220E-09/
      DATA (FF(I),I=1,64)/-5.07092E+00, 4.71960E-03,-3.79851E-03,       TOTAL
     *-3.67309E-06,-6.02439E-01, 1.08490E-04, 5.09287E-04, 5.62210E-07, TOTAL
     * 7.05718E-02, 5.13160E-06,-2.85571E+00,-4.31728E-05, 1.03185E-03, TOTAL
     * 1.05332E-07, 1.04106E-02, 1.60749E-05, 4.18031E-05, 3.32759E-08, TOTAL
     * 1.20113E-01, 1.40486E-05,-3.37993E-05, 5.48340E-09, 9.10815E-02, TOTAL
     *-4.00608E-04, 3.75393E-03,-4.69939E-07,-2.48561E-02, 1.31836E-04, TOTAL
     *-2.67755E-04,-7.60285E-08, 3.04443E-03,-3.28956E-06, 5.82367E-01, TOTAL
     * 5.39496E-06,-6.15261E-04, 4.05316E-08, 1.13546E-02,-4.26493E-06, TOTAL
     *-2.72007E-02, 5.72523E-08,-2.98576E+00, 3.07325E-05, 1.51645E-03, TOTAL
     * 1.25098E-06, 4.07213E-02, 1.05964E-05, 1.04232E-04, 1.77381E-08, TOTAL
     * 1.92781E-01, 2.15734E-05,-1.65741E-05,-1.88683E-09, 2.44803E-01, TOTAL
     * 1.51316E-05,-3.01157E-04, 8.47006E-08, 1.86971E-02,-6.94074E-06, TOTAL
     * 9.13198E-03,-2.38052E-07, 1.28552E-01, 6.92595E-06,-8.36792E-05, TOTAL
     *-6.10021E-08/
      DATA TILTL/99./
C
C     SET UP SOME OF THE INITIAL POSITION VARIABLES
      X=XX(1)
      Y=XX(2)
      Z=XX(3)
      Y2=Y**2
      Z2=Z**2
      R2=X**2+Y2+Z2
C
C     SET MAGNETIC FIELD VARIABLES TO ZERO
      BX=0.
      BY=0.
      BZ=0.
C
C     CHECK TO SEE IF POSITION IS WITHIN REGION OF VALIDITY
      CON=1.
C     IF DISTANCE TOO LARGE TAKE ERROR EXIT
      IF(R2.GT.225.) GO TO 50
C     IF DISTANCE TOO SMALL SET FIELD TO ZERO AND EXIT
      IF(R2.LT.4.)GO TO 40
      IF(R2.LT.6.25) CON=CON*(R2-4.0)/2.25
C
C     IF TILT HAS NOT CHANGED, GO DRECTLY TO FIELD CALCULATION
      IF(TILTL.EQ.TILT)GO TO 6
C     SET UP POWERS OF TILT
      TILTL=TILT
      TT(1)=1
```

```
      TT(2)=TILTL
      TT(3)=TILTL**2
      TT(4)=TILT*TT(3)
C
C     SET UP THE X AND Z COMPONENT TILT INDEPENDENT COEFFICIENTS
      DO  1 I=1,32
      J=(I-1)*2+1
      K=ITA(I)
      A(I)=AA(J)*TT(K)+AA(J+1)*TT(K+2)
      B(I)=BB(J)*TT(K)+BB(J+1)*TT(K+2)
      K=ITC(I)
      E(I)=EE(J)*TT(K)+EE(J+1)*TT(K+2)
      F(I)=FF(J)*TT(K)+FF(J+1)*TT(K+2)
 1    CONTINUE
C
C     SET UP THE Y COMPONENT TILT INDEPENDENT COEFFICIENTS
      DO  2 I=1,22
      J=(I-1)*2+1
      K=ITB(I)
      C(I)=CC(J)*TT(K)+CC(J+1)*TT(K+2)
      D(I)=DD(J)*TT(K)+DD(J+1)*TT(K+2)
 2    CONTINUE
 6    EXPR=EXP(-0.06*R2)
C
C     INITIALIZE THE POINTERS
      II=1
      JJ=1
      KK=1
      XB=1.
C
C     BEGIN SUM OVER X
      DO 30 I=1,5
      YEXB=XB
C
C     BEGIN SUM OVER Y
      DO 20 J=1,3
      IF(I+2*J.GT. 8) GO TO 25
      ZEYEXB=YEXB
      IJK=I+2*J+1
      K=1
C
C     Z LOOP STARTS HERE
 10   BZ=BZ+(E(KK)+F(KK)*EXPR)*ZEYEXB
      KK=KK+1
      BX=BX+(A(II)+B(II)*EXPR )*ZEYEXB
      II=II+1
      IF(IJK .GT. 8) GO TO 15
      BY=BY+(C(JJ)+D(JJ)*EXPR )*ZEYEXB*Y
      JJ=JJ+1
      ZEYEXB=ZEYEXB*Z
      IJK=IJK+1
      K=K+1
      IF(IJK.LE.9.AND.K.LE.5) GO TO 10
 15   YEXB=YEXB*Y2
 20   CONTINUE
 25   XB=XB*X
 30   CONTINUE
C
C     SET UP THE OUTPUT ARRAY, MULTIPLY BY CON. CON IS NORMALY ONE
C     BUT INSIDE OF R=2.5 IT GOES TO ZERO. INSIDE R=2 IT IS ZERO.
```

```
 40   BF(1)=BX*CON
      BF(2)=BY*CON
      BF(3)=BZ*CON
      RETURN
C
C     ERROR EXIT IF OUTSIDE OF R = 15.
 50   WRITE(6,60) XX
 60   FORMAT(4H X= ,E10.3,4H Y= ,E10.3,4H Z= ,E10.3,76H  IS OUTSIDE THE
     *VALID REGION--POWER SERIES DIVERGES BFIELD IS SET TO ZERO  )
      GO TO 40
      END
```

Appendix C

The Dynamic Magnetic Field Routines

This appendix lists the dynamic magnetic subroutines that can be substituted for the tilt dependent routines described in Appendix B. The change the B,L code from a tilt dependent code to a dynamic code requires that all of the routines described in Appendix B be replaced by the routines described in this appendix. The tilt dependent routines require the Universal time and Day of year be set up in COMMON BLOCK BXYZCM. In this version it is not necessary to set up these variables. Instead the standoff distance, strength of the ring and strength of the tail must communicated to the routines via COMMON BLOCK DYNVAR. Note that the dynamic magnetic field program has been given the same name in this version as the tilt dependent routine. This was necessary to enable the user to instantly drop the dynamic routines into the B,L code without modification to the code. Although communicating variable via common block is not considered good programming practice, it saves considerable computer time in this code and makes the change from tilt dependent to dynamic relatively easy. To use INVARM with the dynamic code it is only necessary to set up the following variables in the common block.

COMMON/DYNVAR/SOFFD,SRING,STAIL

SOFFD is the standoff distance of the magnetosphere. The quiet standoff distance is 10.5 earth radii. Acceptable values range between 5 and 11. This value is used to calculate the strength of the magnetopause currents and to scale the size of the magnetopause. This value also scales the size of the tail current system. The ring current is not scaled, since its source is primarily at small radial distances where the field geometry is less affected by the standoff distance.

SRING is the relative strength of the ring current. A value of one utilizes the nominal quiet ring current values built into the basic model. This basic model has a maximum ring depression of 40 nanotesla at $L = 4\ R_e$. If SRING is set to 2 the ring depression will be 80 nanotesla.

STAIL is a tail current strength multiplier. When STAIL is equal to 1.0 then the tail scales with the strength of the magnetopause currents. To weaken the tail from this value use values less than 1.0, to strengthen use values greater than 1.0.

Two support routines are provided with the dynamic magnetic field code, the first is RINGST which calculates the strength of the ring from the standoff distance and D_{st}; the second is STDOFF which calculates the standoff distance from the velocity and number density of the solar wind. The arguments and calling sequence for these support routines are defined on comment cards in the program listing.

```
      SUBROUTINE BMNEXT(XX,B,BMAG)
C*****************************************************************
C  The dynamic variables are communicated via common block
C  DYNVAR
C*****************************************************************
      COMMON/DYNVAR/SOFFD,SRING,STAIL
C
C     PURPOSE
C        TO DETERMINE THE MAIN MAGNETIC FIELD PLUS THE EXTERNAL
C        FIELD
C
C     METHOD
C        DETERMINES THE VECTOR MAGNETIC FIELD IN GEOGRAPHIC
C        COORDINATES USING A SPHERICAL COORDINATE EXPANSION OF THE
C        EARTHS INTERNAL FIELD AND A CARTESIAN COORDINATE EXPANSION
C        OF THE BOUNDARY, TAIL AND RING CURRENT FIELDS IN SOLAR
C        MAGNETIC COORDINATES
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY CONTAINING THE POSITION IN GEOGRAPHIC
C                COORDINATES
C                IF KODE = 1
C                XX(1)=X, XX(2)=Y, XX(3)=Z, WHERE X, Y, Z ARE IN EARTH
C                RADII.  THE DIRECTION OF Z IS ALONG THE EARTHS ROTATION
C                AXIS TOWARDS THE GEOGRAPHIC NORTH POLE.  THE DIRECTION
C                OF X IS TO THE GREENWHICH MERIDIAN IN THE EQUATORIAL
C                PLANE.  THE Y AXIS IS IN THE EQUATORIAL PLANE NORMAL
C                TO X AND Z IN A RIGHT HANDED SENSE.
C                IF KODE = 2
C                XX(1)=R, GEOCENTRIC RADIUS IN EARTH RADII,
C                XX(2)=THETAG, COLATITUDE IN DEGREES,
C                XX(3)=PHIG, LONGITUDE IN DEGREES
C
C     INPUT -- COMMON BLOCK DYNVAR
C  *****Only used for dynamic routine
C        SOFFD   THE STANDOFF DISTANCE OF THE MAGNETOPAUSE.  THE QUIET
C                STANDOFF DISTANCE IS 10.5 EARTH RADII.  ACCEPTABLE
C                VALUES RANGE BETWEEN 5 AND 11.  THIS VALUE IS USED TO
C                CALCULATE THE STRENGTH OF THE MAGNETOPAUSE CURRENTS AND
C                TO SCALE THE SIZE OF THE MAGNETOPAUSE.  THIS VALUE ALSO
C                SCALES THE SIZE OF THE TAIL CURRENT SYSTEM.  THE RING
C                SYSTEM IS NOT SCALED, SINCE ITS SOURCE IS PRIMARILY AT
C                RADIAL DISTANCES.
C
C        SRING   RELATIVE STRENGTH OF THE RING CURRENT.  A VALUE OF ONE
C                UTILIZES THE NOMINAL QUIET RING VALUES BUILT INTO THE
C                BASIC MODEL.  THIS BASIC MODEL HAS A MAXIMUM RING
C                DEPRESSION OF 40 NT AT L=4 RE.  IF SRING IS SET TO 2
C                THE RING DEPRESSION WOULD BE 80 NT.
C
C        STAIL   A TAIL CURRENT STRENGTH MULTIPLIER.  WHEN STAIL IS EQUAL
C                TO 1.0 THEN THE TAIL SCALES WITH THE STRENGTH OF THE
C                MAGNETOPAUSE CURRENTS. TO WEAKEN THE TAIL FROM THIS
C                VALUE USE VALUES LESS THAN 1.0, TO STRENGTHEN USE VALUES
C                GREATER THAN 1.0
C
C
C     INPUT -- COMMON BLOCK BXYZCM
C*******************************************************************
C***    UT      THE CURRENT UNIVERSAL TIME IN HOURS
```

```
C***              only used if tilt dependent B is used
C*******************************************************************
C
C        KODE   A FLOW CONTROL VARIABLE.  KODE EQUAL TO ONE MEANS THAT
C               INPUT AND OUTPUT ARE IN CARTESIAN COORDINATES.  KODE
C               EQUAL TO TWO MEANS THAT INPUT AND OUTPUT ARE SPHERICAL
C               COORDINATES.
C
C*******************************************************************
C***     DAYYR  THE NUMBER OF THE DAY OF YEAR
C***              only used if tilt dependent B is used
C*******************************************************************
C
C        JSW    A FLOW CONTROL VARIABLE. IF JSW IS LESS THAN ZERO, THE
C               THE FIELD IS COMPUTED USING THE INTERNAL FIELD ONLY.
C               IF JSW IS GREATER THAN OR EQUAL TO ZERO THE FIELD
C               WILL BE COMPUTED USING THE INTERNAL PLUS EXTERNAL
C               FIELD.
C        YEAR   THE YEAR USED BY THE INTERNAL MAGNETIC FIELD ROUTINE
C               TO TAKE INTO ACCOUNT THE SECULAR VARIATIONS
C               (E.G. JULY 15, 1964 = 1964.54)
C               NOTE**** YEAR SHOULD BE CHANGED ONLY EVERY FEW DAYS OR
C               MONTHS.  NEW FIELD COEFFICIENTS MUST BE COMPUTED FOR
C               EVERY CHANGE IN YEAR.  THIS COULD CAUSE A LARGE INCREASE
C               IN COMPUTER TIME.  THE EARTHS FIELD CHANGES ONLY ABOUT
C               .001 GAUSS/YEAR AT THE EARTHS SURFACE.
C
C     OUTPUT -- ARGUMENT LIST
C        B      A REAL ARRAY CONTAINING THE COMPONENTS OF THE MAGNETIC
C               FIELD IN GAUSS AT THE CURRENT POSITION AND TIME
C               IF KODE = 1
C               B(1)=BX, B(2)=BY, B(3)=BZ  THE CARTESIAN COMPONENTS
C               OF THE MAGNETIC FIELD IN GEOGRAPHIC COORDINATES
C               IF KODE = 2
C               B(1)=BR, RADIAL COMPONENT OF THE FIELD, POSITIVE IN THE
C               DIRECTION OF INCREASING RADIUS.
C               B(2)=BTHETA, COMPONENT IN LATITUDE, POSITIVE IN THE
C               DIRECTION OF INCREASING COLATITUDE
C               B(3)=BPHI, COMPONENT IN LONGITUDE, POSITIVE IN THE
C               DIRECTION OF INCREASING LONGITUDE.
C        BMAG   THE MAGNITUDE OF THE MAGNETIC FIELD VECTOR IN UNITS OF
C               GAUSS.
C
C     OUTPUT -- COMMON BLOCK BXYZCM
C        XMLAT  THE MAGNETIC LATITUDE AT THE CURRENT POSITION IN RADIANS
C
C     SUBROUTINE CONSTANTS
C        PICON  THE NUMBER OF DEGREES PER RADIAN
C        SIN D  THE SINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        COS D  THE COSINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        C69    COSINE OF 69
C        S69    SINE OF 69
C
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE BXYZMU or BDYNAM
C        SUBROUTINE ANGLE is not needed for BDYNAM
C        SUBROUTINE SPIGRF,FLDCOEF,GETGAU,CINTRP,CONVRT,EXTRAP
C
C     FUNCTIONS STDOFF, and RINGST are provided for to user to help
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C        with the Dynamic B routines
C      VARIABLES
C         AOR     INVERSE OF RADIUS VECTOR (AOR=1./R)
C         BGMX,BGMY,BGMZ  INTERMEDIATE VALUES OF THE MAGNETIC FIELD
C                VECTOR DURING COORDINATE TRANSFORMATION
C         BMX,BMY,BMZ  EXTERNAL MAGNETIC FIELD IN GEOMAGNETIC COORDINATES
C         BP,BR,BT  COMPONENTS OF INTERNAL FIELD IN SPHERICAL COORDINATES
C                BP IS LONGITUDINAL COMPONENT
C                BR IS RADIAL COMPONENT
C                BT IS LATITUDINAL COMPONENT
C         BX,BY,BZ  CARTESIAN COMPONENT OF EXTERNAL FIELD IN GEOGRAPHIC
C                COORDINATES
C         CP      COSINE OF COLATITUDE
C         CPS     COSINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C                GEOMAGNETIC COORDINATES
C         CT      COSINE OF GEOGRAPHIC LONGITUDE
C         DAYLST LAST DAY FOR WHICH TILT AND HOUR ANGLE WERE UPDATED
C         NMAX    MAXIMUM NUMBER OF TERMS USED BY INTERNAL FIELD ROUTINE
C                SET UP BY INTERNAL FIELD ROUTINE
C         PHIG    GEOGRAPHIC COLATITUDE
C         R       RADIUS VECTOR TO POSITION POINT
C         R2      R**2
C         SP      SINE OF COLATITUDE
C         SPS     SINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C                GEOMAGNETIC COORDINATES
C         ST      SINE OF LONGITUDE
C         THETAG GEOGRAPHIC LONGITUDE
C         TILT    TILT OF THE DIPOLE AXIS
C         UTLST   LAST UNIVERSAL TIME FOR WHICH TILT AND HOUR ANGLE WERE
C                UPDATED
C         X       A REAL ARRAY HOLDING THE POSITION VECTOR IN SOLAR
C                MAGNETIC COORDINATES
C         XP,YP,ZP  POSITION VECTOR IN GEOMAGNETIC COORDINTES
C         XPP,YPP INTERMEDIATE POSITION COMPONENT DURING COORDINATE
C                TRANSFORMATION
C         YEARI   TRANSMITS THE YEAR TO THE INTERNAL FIELD ROUTINE
C
C      VERSION 10/25/77
C      FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER AT MCDONNELL
C      DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON BEACH CALIF.
C      PHONE (714) 896-3231.
C
       DIMENSION X(3),B(3),XX(3)
       COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
       COMMON /GCOM/ ST,CT,SP,CP,AOR,BT,BP,BR,NMAX,YEARI
       DATA PICON/57.29577951/,SIND,COSD/.2027872954,.9792228106/,
      *S69,C69/.9335804265,.3583679495/,UTLST,DAYLST/2*123456./
C
C      UPDATE THE ROTATION HOUR ANGLE AND TILT ANGLE IF THE UNIVERSAL
C      TIME OR DAY OF YEAR HAS CHANGED SINCE THE LAST CALL
C
       IF(UT.EQ.UTLST.AND.DAYYR.EQ.DAYLST) GO TO 1
       UTLST=UT
       DAYLST=DAYYR
       CALL ANGLE (TILT,SPS,CPS)
 1     IF(KODE.GT.1) GO TO 3
C
C      DETERMINE THE SPHERICAL COORDINATES OF POSITION IF CARTESIAN
C      COORDINATES WERE ENTERED
C
```

```
      X(1)=XX(1)
      X(2)=XX(2)
      X(3)=XX(3)
      R2=X(1)**2+X(2)**2
      R=SQRT(X(3)**2+R2)
      R2=SQRT(R2)
      CT=X(3)/R
      ST=R2/R
      CP=X(1)/R2
      SP=X(2)/R2
      GO TO 5
C
C     DETERMINE THE CARTESIAN COORDINATES OF POSITION IF SPHERICAL
C     COORDINATES WERE ENTERED
C
 3    R=XX(1)
      THETAG=XX(2)/PICON
      PHIG=XX(3)/PICON
      CT=COS(THETAG)
      ST=SIN(THETAG)
      CP=COS(PHIG)
      SP=SIN(PHIG)
      X(1)=R*ST*CP
      X(2)=R*ST*SP
      X(3)=R*CT
 5    BX=0.
      BY=0.
      BZ=0.
C
C     IF THE EXTERNAL MAGNETIC FIELD IS TO BE USED IN THE COMPUTATION,
C     COMPUTE THE SOLAR MAGNETIC COORDINATES
C
      IF(JSW.LT.0) GO TO 9
C
C     FIRST ROTATION IS ABOUT THE Z-AXIS TROUGH AN ANGLE OF 291 DEGREES
C     (THE LONGITUDE OF THE MAGNETIC NORTH POLE)
C
      XPP=X(1)*C69-X(2)*S69
      YPP= X(1)*S69+X(2)*C69
C
C     SECOND ROTATION IS ABOUT THE NEW Y-AXIS THROUGH AN ANGLE OF 11.7
C     DEGREES (THE COLATITUDE OF THE MAGNETIC NORTH POLE)
C
      ZP=XPP*SIND+X(3)*COSD
      XP=XPP*COSD-X(3)*SIND
      YP=YPP
C
C     ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR ANGLE OF
C     THE SUN FROM THE PRIME MAGNETIC MERIDIAN (NEGATIVE ROTATION)
C
      X(1)=XP*CPS-YP*SPS
      X(2)=XP*SPS+YP*CPS
      X(3)=ZP
C
C     DETERMINE THE EXTERNAL MAGNETIC FIELD USING A TILT DEPENDENT
C     MAGNETIC FIELD
C
C***************************************************************
C***************************************************************
C******this calls the tilt dependent routine.  To activate remove
```

```
C       the C from the start of the line and comment out the call
C       to BDYNAM
c       CALL BXYZMU(X,B,TILT)
C*****************************************************************
C*****************************************************************
C
      CALL BDYNAM(X,B,SOFFD,SRING,STAIL)
C
C     THE CARTESIAN COMPONENTS OF THE FIELD ARE IN SOLAR MAGNETIC
C     COORDINATES. THE COMPONENTS ARE NEEDED IN THE GEOGRAPHIC
C     COORDINATE SYSTEM
C
C     FIRST ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR
C     ANGLE OF THE SUN TO THE PRIME MAGNETIC MERIDIAN
C     (POSITIVE ROTATION) PUTS RESULTS INTO GEOMAGNETIC COORDINATES
C
      BMX=B(1)*CPS+B(2)*SPS
      BMY=-B(1)*SPS+B(2)*CPS
      BMZ=B(3)
C
C     SECOND ROTATION IS ABOUT THE MAGNETIC Y-AXIS THOUGH -11.7 DEGREES
C     COLATITUDE
C
      BGMX=BMX*COSD+BMZ*SIND
      BGMY=BMY
      BGMZ=-BMX*SIND+BMZ*COSD
C
C     THIRD ROTATION IS ABOUT THE NEW Z-AXIS THROUGH -291 DEGREES
C
      BX=BGMX*C69+BGMY*S69
      BY=-BGMX*S69+BGMY*C69
      BZ=BGMZ
C
C     DETERMINE THE MAIN FIELD
C
 9    CONTINUE
      AOR=1./R
      YEARI=YEAR
      CALL SPIGRF
      IF(KODE.GT.1) GO TO 10
C
C     IF THE OUTPUT IS TO BE IN CARTESIAN GEOGRAPHIC COORDINATES CONVERT
C     THE MAIN MAGNETIC FIELD AND ADD
C
      B(1)=(BX+CP*(ST*BR+CT*BT)-SP*BP)*0.00001
      B(2)=(BY+SP*(ST*BR+CT*BT)+CP*BP)*0.00001
      B(3)=(BZ+CT*BR-ST*BT)*0.00001
      GO TO 20
C
C     IF OUTPUT IS TO BE IN SPHERICAL GEOGRAPHIC CONVERT THE EXTERNAL
C     FIELD AND ADD
C
 10   B(1)=(BR+(BX*CP+BY*SP)*ST+BZ*CT)*0.00001
      B(2)=(BT+(BX*CP+BY*SP)*CT-BZ*ST)*0.00001
      B(3)=(BP+BY*CP-BX*SP)*0.00001
C
C     DETERMINE THE MAGNITUDE OF THE FIELD VECTOR
C
 20   BMAG=SQRT(B(1)**2+B(2)**2+B(3)**2)
      RETURN
```

END

```
      SUBROUTINE BDYNAM(XX,BB,SOFFD,SRING,STAIL)
C  VERSION 5/13/88
C  DEVELOPED MCDONNELL DOUGLAS
C  FOR INFORMATION CALL KARL PFITZER (714) 896-3231
C
C     PURPOSE
C        CALCULATE THE TOTAL EXTERNAL MAGNETIC FIELD DURING DISTURBED
C        TIMES
C
C     METHOD
C        CALLS THE EXTERNAL QUIET TIME SUBROUTINES AND COMBINES THEM
C        ACCORDING TO THE DYNAMIC SCALING ALGORITHMS
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C                FIELD IS TO BE DETERMINED.  XX(1), XX(2), XX(3) ARE
C                RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC COORDINATES
C                IN EARTH RADII.  Z IS ALONG THE EARTHS NORTH DIPOLE
C                AXIS. X IS PERPENDICULAR TO Z AND IN THE PLANE
C                CONTAINING THE Z AXIS AND THE SUN-EARTH LINE (X IS
C                POSITIVE IN THE SOLAR DIRECTION).  Y IS PERPENDICULAR
C                TO X AND Z AND X Y Z FORM A RIGHT HANDED COORDINATE
C                SYSTEM.
C
C        SOFFD   THE STANDOFF DISTANCE OF THE MAGNETOPAUSE.  THE QUIET
C                STANDOFF DISTANCE IS 10.5 EARTH RADII.  ACCEPTABLE
C                VALUES RANGE BETWEEN 6 AND 11.  THIS VALUE IS USED TO
C                CALCULATE THE STRENGTH OF THE MAGNETOPAUSE CURRENTS AND
C                TO SCALE THE SIZE OF THE MAGNETOPAUSE.  THIS VALUE ALSO
C                SCALES THE SIZE OF THE TAIL CURRENT SYSTEM.  THE RING
C                SYSTEM IS NOT SCALED, SINCE ITS SOURCE IS PRIMARILY AT
C                RADIAL DISTANCES.
C
C        SRING   RELATIVE STRENGTH OF THE RING CURRENT.  A VALUE OF ONE
C                UTILIZES THE NOMINAL QUIET RING VALUES BUILT INTO THE
C                BASIC MODEL.  THIS BASIC MODEL HAS A MAXIMUM RING
C                DEPRESSION OF 40 NT AT L=4 RE.  IF SRING IS SET TO 2
C                THE RING DEPRESSION WOULD BE 80 NT.
C
C        STAIL   A TAIL CURRENT STRENGTH MULTIPLIER.  WHEN STAIL IS EQUAL
C                TO 1.0 THEN THE TAIL SCALES WITH THE STRENGTH OF THE
C                MAGNETOPAUSE CURRENTS. TO WEAKEN THE TAIL FROM THIS
C                VALUE USE VALUES LESS THAN 1.0, TO STRENGTHEN USE VALUES
C                GREATER THAN 1.0
C
C     OUTPUT -- ARGUMENT LIST
C        B       A REAL ARRAY CONTAINING THE X, Y, AND Z COMPONENTS
C                OF THE EARTHS TOTAL MAGNETIC FIELD IN SOLAR MAGNETIC
C                COORDINATES.  B(1), B(2) AND B(3) ARE THE BX, BY, AND BZ
C                COMPONENTS.  THE UNITS ARE GAUSS.
C
      DIMENSION XX(3),BB(3),XXX(3),BM(3),BR(3),BT(3),xxxx(3)
      DATA CON/0.025/
C   CALCULATE MAGNETOPUSE SCALE FACTOR
      SCL=10.5/SOFFD
C   CALCULATE STRENGTH OF MAGNETOPAUSE CURRENTS
      STRMAG=SCL**3
C   SET STRENGTH OF RING AND TAIL
      STRRIN=Sring
      STRTAI=STAIL*STRMAG
```

```
C    CALCULATE SCALED DISTANCES
       DO 10 I=1,3
       XXX(I)=XX(I)*SCL
       XXXX(I)=XX(I)*1.0
10     CONTINUE
C    CALL THE QUIET TIME SUBROUTINE
       CALL BFMAGP(XXX,BM)
       CALL BFRING(XXXX,BR)
       CALL BFTAIL(XXX,BT)
C    COMBINE THE COMPONENTS OF THE MAGNETIC FIELD ACCORDING TO THEIR
C    RELATIVE STRENGTHS
       DO 20 I=1,3
       BB(I)=STRMAG*BM(I)+STRRIN*BR(I)+STRTAI*BT(I)
20     CONTINUE
       RETURN
       END
```

```
      SUBROUTINE BFMAGP(XX,BB)
C  VERSION 5/13/88
C  DEVELOPED MCDONNELL DOUGLAS
C  FOR INFORMATION CALL KARL PFITZER (714) 896-3231
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C                FIELD IS TO BE DETERMINED.  XX(1), XX(2), XX(3) ARE
C                RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC COORDINATES
C                IN EARTH RADII.  Z IS ALONG THE EARTHS NORTH DIPOLE
C                AXIS. X IS PERPENDICULAR TO Z AND IN THE PLANE
C                CONTAINING THE Z AXIS AND THE SUN-EARTH LINE (X IS
C                POSITIVE IN THE SOLAR DIRECTION).  Y IS PERPENDICULAR
C                TO X AND Z AND X Y Z FORM A RIGHT HANDED COORDINATE
C                SYSTEM.
C
C     OUTPUT -- ARGUMENT LIST
C        BB      A REAL ARRAY CONTAINING THE X, Y, AND Z COMPONENTS
C                OF THE EARTHS TOTAL MAGNETIC FIELD IN SOLAR MAGNETIC
C                COORDINATES.  BB(1), BB(2) AND BB(3) ARE THE BX, BY,
C                AND BZ COMPONENTS.  THE UNITS ARE NANOTESLA.
C
C THIS SUBROUTINE CONTAINS NEW, REFITTED COEFFICIENTS FOR COMPUTING
C ALL THE B-MAGNETOPAUSE COMPONENTS.
C THE FORM OF THE EXPANSION IS GIVEN IN THE NEXT FEW STATEMENTS
C
C BMPX=(1 + X + X**2 + X**3 + X**4 + 1/(10-(X-2)**2))*(1 + Y**2 + Y**4)*
C      (Z + Z**3 + Z**5).
C BMPY=(1 + X + X**2 + X**3 + X**4)*(Y + Y**3 + Y**5)*(Z + Z**3 + Z**5).
C BMPZ=(1 + X + X**2 + X**3 + X**4 + 1/(15-X) + 1/((30-X)**2))*
C      (1 + Y**2 + Y**4)*(1 + Z**2 + Z**4).
C
C COEFFICIENTS COMPUTED FROM COMBINED OLSON DATA-SETS BOUNDARY.DAT AND
C BOUND.DAT, FOR Y>0 AND Z>0 ONLY, IE. A TOTAL OF 1009 DATA POINTS.
C 2 EXTRA  EXTRAPOLATED POINTS ADDED FOR Z-COEFF, TO IMPROVE FIT,
C NAMELY X=10,Y=Z=0,BZ=29 AND X=11,Y=Z=0,BZ=30.25.
C
C***PRELIMINARY ROUTINE
C***VALID TO APPROXIMATELY -60 RE
C***MAGNETIC FIELD FROM MAGNETOPAUSE CURRENTS ONLY
C
      DIMENSION XX(3),BB(3),A(54),B(45),C(63),C1(30),C2(33)
      DIMENSION X(5),Y(5),Z(5)
      EQUIVALENCE (C(1),C1(1)),(C(31),C2(1))
      DATA A/
     *  0.113275039E+01, 0.354408138E-01,-0.152252289E-02,
     * -0.683306571E-04,-0.642841428E-06,-0.121504674E-01,
     * -0.839622808E-03,-0.167520029E-04,-0.385962942E-07,
     *  0.107674747E-08, 0.558984066E-04, 0.551508083E-05,
     *  0.206288036E-06, 0.335316730E-08, 0.198413126E-10,
     * -0.545824692E-02,-0.264107861E-03, 0.143533146E-05,
     *  0.195177861E-06, 0.207546358E-08, 0.211199178E-03,
     *  0.220245929E-04, 0.860991804E-06, 0.145349395E-07,
     *  0.886173426E-10,-0.949615014E-06,-0.110830563E-06,
     * -0.477998707E-08,-0.873645670E-10,-0.569051859E-12,
     *  0.271760982E-04, 0.266707661E-05, 0.994617153E-07,
     *  0.167023062E-08, 0.104617062E-10,-0.989193381E-06,
     * -0.113236254E-06,-0.482686247E-08,-0.880319914E-10,
     * -0.575385009E-12, 0.487020380E-08, 0.586310778E-09,
     *  0.260182431E-10, 0.488435735E-12, 0.326678627E-14,
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     *  0.193470073E+01, 0.402453184E+00,-0.193471275E-02,
     *  0.682263300E-01,-0.576195028E-02, 0.237557251E-04,
     * -0.529665092E-03, 0.255710365E-04,-0.120115033E-06/
C
      DATA B/
     * -0.519952811E-01,-0.230140495E-02, 0.146173188E-03,
     *  0.809832090E-05, 0.888401672E-07,-0.370911323E-03,
     * -0.101231737E-03,-0.742647399E-05,-0.196170248E-06,
     * -0.165503899E-08, 0.150949325E-05, 0.308240260E-06,
     *  0.195390104E-07, 0.472441419E-09, 0.375989214E-11,
     * -0.422217818E-04,-0.621468353E-04,-0.620102765E-05,
     * -0.189322407E-06,-0.172039538E-08, 0.445292017E-05,
     *  0.118324999E-05, 0.855768008E-07, 0.223059815E-08,
     *  0.183677951E-10,-0.550030643E-08,-0.150351465E-08,
     * -0.107031245E-09,-0.268793755E-11,-0.205845354E-13,
     *  0.813519478E-06, 0.279971147E-06, 0.227601529E-07,
     *  0.643000209E-09, 0.561745876E-11,-0.983297266E-08,
     * -0.265465072E-08,-0.194798427E-09,-0.513382522E-11,
     * -0.420117906E-13,-0.469398392E-11,-0.543405219E-12,
     * -0.121854998E-13,-0.483310746E-16,-0.429469692E-17/
C
      DATA C1/
     *  0.406363373E+02, 0.291153884E+01, 0.991215929E-01,
     *  0.161603605E-02, 0.994476977E-05,-0.566497850E+01,
     * -0.346289247E+00,-0.102486340E-01,-0.153071058E-03,
     * -0.892381365E-06, 0.182735808E-01, 0.106282183E-02,
     *  0.311990625E-04, 0.464014079E-06, 0.269492229E-08,
     * -0.102119482E+01,-0.649643913E-01,-0.205774955E-02,
     * -0.323610875E-04,-0.195236396E-06, 0.531459488E-01,
     *  0.324825896E-02, 0.991819543E-04, 0.152400162E-05,
     *  0.907312536E-08,-0.132267553E-03,-0.871756401E-05,
     * -0.262251859E-06,-0.395617938E-08,-0.232419934E-10/
      DATA C2/
     *  0.144323579E-02, 0.799393092E-04, 0.322526876E-05,
     *  0.596131713E-07, 0.395406097E-09,-0.839159111E-05,
     *  0.564246250E-06,-0.212045990E-07,-0.866837990E-09,
     * -0.746255575E-11,-0.685688633E-06,-0.523054773E-07,
     * -0.130326583E-08,-0.157964718E-10,-0.759061461E-13,
     *  0.836994324E+02,-0.609500999E+02, 0.100208335E+00,
     * -0.688268995E+01, 0.397136599E+00,-0.250137411E-02,
     * -0.594024621E-01, 0.457714684E-02,-0.449951913E-04,
     * -0.273244004E+05, 0.875882129E+04,-0.227706509E+02,
     *  0.129124341E+04,-0.715722046E+02, 0.266965359E+00,
     *  0.240404391E+01,-0.269608498E+00, 0.332747493E-02/
C
      XN=XX(1)
      YN=XX(2)
      ZN=XX(3)
      DO 1 I=1,5
      X(I)=XN
      Y(I)=YN
      Z(I)=ZN
      XN=XN*XX(1)
      YN=YN*XX(2)
      ZN=ZN*XX(3)
    1 CONTINUE
      FX1=1./(10.+(X(1)-2.)**2)
      XF1=1./(15.-X(1))
      XF2=1./((30.-X(1))**2)
C
```

```
      BB(1)=Z(1)*(A(1)+A(2)*X(1)+A(3)*X(2)+A(4)*X(3)+A(5)*X(4))+
     * Z(3)*(A(6)+A(7)*X(1)+A(8)*X(2)+A(9)*X(3)+A(10)*X(4))+
     * Z(5)*(A(11)+A(12)*X(1)+A(13)*X(2)+A(14)*X(3)+A(15)*X(4))+
     * Y(2)*Z(1)*(A(16)+A(17)*X(1)+A(18)*X(2)+A(19)*X(3)+A(20)*X(4))+
     * Y(2)*Z(3)*(A(21)+A(22)*X(1)+A(23)*X(2)+A(24)*X(3)+A(25)*X(4))+
     * Y(2)*Z(5)*(A(26)+A(27)*X(1)+A(28)*X(2)+A(29)*X(3)+A(30)*X(4))+
     * Y(4)*Z(1)*(A(31)+A(32)*X(1)+A(33)*X(2)+A(34)*X(3)+A(35)*X(4))+
     * Y(4)*Z(3)*(A(36)+A(37)*X(1)+A(38)*X(2)+A(39)*X(3)+A(40)*X(4))+
     * Y(4)*Z(5)*(A(41)+A(42)*X(1)+A(43)*X(2)+A(44)*X(3)+A(45)*X(4))+
     * FX1*(A(46)*Z(1)+A(47)*Z(3)+A(48)*Z(5))+FX1*Y(2)*(A(49)*Z(1)+
     * A(50)*Z(3)+A(51)*Z(5))+FX1*Y(4)*(A(52)*Z(1)+A(53)*Z(3)+A(54)*
     * Z(5))
C
      BB(2)=Z(1)*Y(1)*(B(1)+B(2)*X(1)+B(3)*X(2)+B(4)*X(3)+B(5)*X(4))+
     * Z(3)*Y(1)*(B(6)+B(7)*X(1)+B(8)*X(2)+B(9)*X(3)+B(10)*X(4))+
     * Z(5)*Y(1)*(B(11)+B(12)*X(1)+B(13)*X(2)+B(14)*X(3)+B(15)*X(4))+
     * Y(3)*Z(1)*(B(16)+B(17)*X(1)+B(18)*X(2)+B(19)*X(3)+B(20)*X(4))+
     * Y(3)*Z(3)*(B(21)+B(22)*X(1)+B(23)*X(2)+B(24)*X(3)+B(25)*X(4))+
     * Y(3)*Z(5)*(B(26)+B(27)*X(1)+B(28)*X(2)+B(29)*X(3)+B(30)*X(4))+
     * Y(5)*Z(1)*(B(31)+B(32)*X(1)+B(33)*X(2)+B(34)*X(3)+B(35)*X(4))+
     * Y(5)*Z(3)*(B(36)+B(37)*X(1)+B(38)*X(2)+B(39)*X(3)+B(40)*X(4))+
     * Y(5)*Z(5)*(B(41)+B(42)*X(1)+B(43)*X(2)+B(44)*X(3)+B(45)*X(4))
C
       BB(3)=C(1)+C(2)*X(1)+C(3)*X(2)+C(4)*X(3)+C(5)*X(4)+
     * Z(2)*(C(6)+C(7)*X(1)+C(8)*X(2)+C(9)*X(3)+C(10)*X(4))+
     * Z(4)*(C(11)+C(12)*X(1)+C(13)*X(2)+C(14)*X(3)+C(15)*X(4))+
     * Y(2)*(C(16)+C(17)*X(1)+C(18)*X(2)+C(19)*X(3)+
     * C(20)*X(4))+Y(2)*Z(2)*(C(21)+C(22)*X(1)+C(23)*X(2)+
     * C(24)*X(3)+C(25)*X(4))+Y(2)*Z(4)*(C(26)+C(27)*X(1)+
     * C(28)*X(2)+C(29)*X(3)+C(30)*X(4))+Y(4)*(C(31)+
     * C(32)*X(1)+C(33)*X(2)+C(34)*X(3)+C(35)*X(4))
      BB(3)=BB(3)+
     * Y(4)*Z(2)*(C(36)+C(37)*X(1)+C(38)*X(2)+C(39)*X(3)+C(40)*X(4))+
     * Y(4)*Z(4)*(C(41)+C(42)*X(1)+C(43)*X(2)+C(44)*X(3)+
     * C(45)*X(4))+XF1*(C(46)+C(47)*Z(2)+C(48)*Z(4)+
     * C(49)*Y(2)+C(50)*Y(2)*Z(2)+C(51)*Y(2)*Z(4)+C(52)*Y(4)+
     * C(53)*Y(4)*Z(2)+C(54)*Y(4)*Z(4))+XF2*(C(55)+C(56)*Z(2)+
     * C(57)*Z(4)+C(58)*Y(2)+C(59)*Y(2)*Z(2)+C(60)*Y(2)*Z(4)+C(61)*Y(4)+
     * C(62)*Y(4)*Z(2)+C(63)*Y(4)*Z(4))
       RETURN
       END
```

```
      SUBROUTINE BFTAIL(XX,BB)
C  VERSION 5/13/88
C  DEVELOPED MCDONNELL DOUGLAS
C  FOR INFORMATION CALL KARL PFITZER (714) 896-3231
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C                FIELD IS TO BE DETERMINED.  XX(1), XX(2), XX(3) ARE
C                RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC COORDINATES
C                IN EARTH RADII.  Z IS ALONG THE EARTHS NORTH DIPOLE
C                AXIS. X IS PERPENDICULAR TO Z AND IN THE PLANE
C                CONTAINING THE Z AXIS AND THE SUN-EARTH LINE (X IS
C                POSITIVE IN THE SOLAR DIRECTION).  Y IS PERPENDICULAR
C                TO X AND Z AND X Y Z FORM A RIGHT HANDED COORDINATE
C                SYSTEM.
C
C     OUTPUT -- ARGUMENT LIST
C        BB      A REAL ARRAY CONTAINING THE X, Y, AND Z COMPONENTS
C                OF THE EARTHS TOTAL MAGNETIC FIELD IN SOLAR MAGNETIC
C                COORDINATES.  BB(1), BB(2) AND BB(3) ARE THE BX, BY,
C                AND BZ COMPONENTS.  THE UNITS ARE NANOTESLA.
C
C  THIS IS THE EXPANSION FOR THE TAIL CURRENT SYSTEM
C  THE EXPANSION IS VALID FROM THE SUBSOLAR POIN TO -60 RE
C  THE EXPANSION IS BASES ON A FIT TO VALUES CALCUALTED USING THE
C  WIRE LOOP TAIL SYSTEM
      DIMENSION XX(3),BB(3),A(65),B(17),C(39)
      DIMENSION X(5),Y(5),Z(5)
      DATA A/
     *-.118386794E-12, .260137167E+01, .408016277E-12,-.306063863E+00,
     * .852659791E-13, .848404600E-14,-.568097241E-02,-.601368497E-14,
     *-.336276159E-13,-.676779936E-15,-.110762251E-02,-.150912058E-15,
     *-.477506548E-14,-.805245718E-02,-.130105300E-14, .442299435E-16,
     *-.432185140E-04,-.520612496E-01,-.918209408E-04,-.686114562E-03,
     * .275041492E-04, .235864029E-15,-.628394374E-04,-.236539414E-16,
     * .379298441E-18,-.109452698E-14,-.163675727E-16,-.766199004E-04,
     *-.110519916E-15,-.111417355E-17, .311215382E-17,-.605957952E-06,
     *-.609414361E+01, .207037106E-12, .130315144E+00,-.250115110E-13,
     * .325228977E+00, .169606672E-01,-.131084126E-14, .232305257E-03,
     * .254138418E-01,-.585580678E-03, .344211139E-16, .268904941E-05,
     * .561115936E-01,-.855121118E-15, .577135898E-03,-.389637036E-04,
     * .531094438E-18, .517250317E-14, .163439821E-17, .280008382E-15,
     * .311491125E-17, .165293989E-02,-.149174308E-16, .406457779E-05,
     *-.415855886E-06, .127866736E-03,-.106070848E-04, .105524883E-17,
     * .293942950E-05,-.417367450E-06, .134032750E-04,-.139506296E-18,
     * 0.0/
      DATA B/
     *-.323149328E-01, .430535014E-02, .115661689E-03,-.486002660E-04,
     *-.102777234E-04,-.489864422E-05,-.356884232E-04,-.334316125E-07,
     * .122456608E+00, .202317315E-01,-.487990709E-03, .338684854E-04,
     *-.511755985E-04, .119096933E-04, .609353153E-03,-.243627124E-05,
     * 0.0/
      DATA C/
     * .318422091E+00, .154017442E+00, .337581827E-01, .436882397E-01,
     *-.153732787E-03, .362817457E-02, .179382198E-03,-.394772816E-05,
     *-.193942567E-01,-.263603775E-04,-.314364082E-04,-.103110548E-02,
     * .386165884E-06,-.301272556E-06,-.102838611E-03,-.725608973E-04,
     *-.893564810E-05,-.200670765E-05,-.805631807E-05,-.217861072E+02,
     *-.219688864E+01, .178558432E+00, .144137907E-01,-.293171667E-04,
     * .178727330E-01, .846703874E-02, .292860242E-04,-.583591628E+00,
```

```
     * .177991433E-02, .253212943E-02,-.629907297E-01, .669977751E-04,
     * .141706101E-03,-.334067698E-03, .122648694E-03,-.259383966E-07,
     * .252027517E-04,-.212223753E-02,
     * 0.0/
      XN=XX(1)
      YN=XX(2)
      ZN=XX(3)
      R2=XN*XN+YN*YN+ZN*ZN
      R=SQRT(R2)
      DO 1 I=1,5
      X(I)=XN
      Y(I)=YN
      Z(I)=ZN
      XN=XN*XX(1)
      YN=YN*XX(2)
      ZN=ZN*XX(3)
    1 CONTINUE
      R22=SQRT((22.-X(1))**2+Y(2)+Z(2))
      EXPC=EXP(X(1)/15.)
      TANZR=TANH(Z(1))*(1.-TANH((8.-R)/5.))
      EXPR=EXP(-(R22-29)**2/60.)
      BB(1)=(A( 2)*Z( 1)+A( 4)*X( 1)*Z( 1)
     *+A( 7)*Y( 2)*Z( 1)
     *+A(11)*X( 1)*Y( 2)*Z( 1)
     *+A(17)*X( 2)*Y( 2)*Z( 1)+A(18)*Z( 3)+A(19)*Y( 2)*Z( 3)+A(20)*X( 1)
     **Z( 3)+A(21)*X( 2)*Z( 3)+A(23)*X( 3)*Z( 1)
     *+A(28)*Y( 4)*Z( 1)
     *+A(32)*X( 4)*Z( 1))*EXPC
     *+(0.0 +A(33)+A(35)*X( 1)+A(37)*Z( 2)
     *+A(38)*Y( 2)+A(40)*Y( 2)*Z( 2)+A(41)*X( 1)*Z( 2)
     *+A(42)*X( 1)*Y( 2)+A(44)*X( 1)*Y( 2)*Z( 2)
     *+A(45)*X( 2)+A(47)*X( 2)*Z( 2)+A(48)*X( 2)*Y( 2)
     *+A(54)*X( 3)+A(56)*X( 3)
     **Z( 2)+A(57)*X( 3)*Y( 2)+A(58)*Z( 4)+A(59)*Y( 4)
     *+A(61)*X( 1)*Z( 4)+A(62)*X( 1)*Y( 4)+A(63)*X( 4))*TANZR
      BB(2)=(B( 1)*Y( 1)*Z( 1)+B( 2)*X( 1)*Y( 1)*Z( 1)+B( 3)*Y( 1)*Z( 3)
     *+B( 4)*Y( 3)*Z( 1)+B( 5)*X( 1)*Y( 1)*Z( 3)+B( 6)*X( 1)*Y( 3)*Z( 1)
     *+B( 7)*X( 2)*Y( 1)*Z( 1)+B( 8)*X( 3)*Y( 1)*Z( 1))*EXPC
     *+(0.0 +B( 9)*Y( 1)
     **Z( 1)+B(10)*X( 1)*Y( 1)*Z( 1)+B(11)*Y( 1)*Z( 3)+B(12)*Y( 3)*Z( 1)
     *+B(13)*X( 1)*Y( 1)*Z( 3)+B(14)*X( 1)*Y( 3)*Z( 1)+B(15)*X( 2)*Y( 1)
     **Z( 1)+B(16)*X( 3)*Y( 1)*Z( 1))*EXPR
      BB(3)=(C( 1)+C( 2)*X( 1)+C( 3)*Z( 2)+C( 4)*Y( 2)+C( 5)*Y( 2)*Z( 2)
     *+C( 6)*X( 1)*Z( 2)+C( 7)*X( 1)*Y( 2)+C( 8)*X( 1)*Y( 2)*Z( 2)+C( 9)
     **X( 2)+C(10)*X( 2)*Z( 2)+C(11)*X( 2)*Y( 2)+C(12)*X( 3)+C(13)*X( 3)
     **Z( 2)+C(14)*X( 3)*Y( 2)+C(15)*Z( 4)+C(16)*Y( 4)+C(17)*X( 1)*Z( 4)
     *+C(18)*X( 1)*Y( 4)+C(19)*X( 4))*EXPC
     *+(0.0 +C(20)+C(21)*X( 1)+C(22)*Z( 2)
     *+C(23)*Y( 2)+C(24)*Y( 2)*Z( 2)+C(25)*X( 1)*Z( 2)+C(26)*X( 1)*Y( 2)
     *+C(27)*X( 1)*Y( 2)*Z( 2)+C(28)*X( 2)+C(29)*X( 2)*Z( 2)+C(30)*X( 2)
     **Y( 2)+C(31)*X( 3)+C(32)*X( 3)*Z( 2)+C(33)*X( 3)*Y( 2)+C(34)*Z( 4)
     *+C(35)*Y( 4)+C(36)*X( 1)*Z( 4)+C(37)*X( 1)*Y( 4)+C(38)*X( 4))*EXPR
      RETURN
      END
```

```
      SUBROUTINE BFRING(XX,BB)
C   VERSION 5/13/88
C   DEVELOPED MCDONNELL DOUGLAS
C   FOR INFORMATION CALL KARL PFITZER (714) 896-3231
C
C      INPUT -- ARGUMENT LIST
C         XX       A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C                  FIELD IS TO BE DETERMINED.  XX(1), XX(2), XX(3) ARE
C                  RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC COORDINATES
C                  IN EARTH RADII.  Z IS ALONG THE EARTHS NORTH DIPOLE
C                  AXIS. X IS PERPENDICULAR TO Z AND IN THE PLANE
C                  CONTAINING THE Z AXIS AND THE SUN-EARTH LINE (X IS
C                  POSITIVE IN THE SOLAR DIRECTION).  Y IS PERPENDICULAR
C                  TO X AND Z AND X Y Z FORM A RIGHT HANDED COORDINATE
C                  SYSTEM.
C
C      OUTPUT -- ARGUMENT LIST
C         BB       A REAL ARRAY CONTAINING THE X, Y, AND Z COMPONENTS
C                  OF THE EARTHS TOTAL MAGNETIC FIELD IN SOLAR MAGNETIC
C                  COORDINATES.  BB(1), BB(2) AND BB(3) ARE THE BX, BY,
C                  AND BZ COMPONENTS.  THE UNITS ARE NANOTESLA.
C
C   THIS SUBROUTINE CALCULATES THE FIELD FROM THE RING CURRENT SYSTEM.
C   THE EXPANSION IS A FIT TO VALUES CALULATED FROM THE WIRE RING
C   CURRENT MODEL.  THE EXPANSION IS VALID FROM THE SUBSOLAR POINT
C   TO -60 RE.
      DIMENSION XX(3),BB(3),A(29),B(17),C(39)
      DIMENSION X(5),Y(5),Z(5)
      DATA A/
     * .937029737E+00,-.734269078E+00,-.125896726E-01,-.843388063E-02,
     * .756104711E-04, .294507011E-02,-.719118601E-03,-.177154663E-01,
     * .104113319E-03,-.339745485E-04, .324439655E-03, .492786378E-04,
     *-.100821105E-04, .109966887E-04, .119616338E+00, .403556177E+01,
     *-.363651494E-01,-.337286459E-01,-.908902973E-04,-.980450316E-01,
     *-.220988518E+00,-.244671475E+00,-.977974501E-03, .311933785E-01,
     *-.249204900E+00, .825058070E-03, .464195892E-02, .223651513E-01,
     * 0.0/
      DATA B/
     *-.908641389E+00,-.249680217E-01, .443512048E-02,-.124215709E-03,
     * .211679921E-03,-.368134800E-04, .547288643E-03, .164845371E-04,
     * .407818714E+01,-.129156231E+00,-.940633654E-01,-.220684438E+00,
     * .878070158E-04, .174193445E-01,-.223040987E+00, .151981648E-01,
     * 0.0/
      DATA C/
     *-.381390073E+02,-.362173083E+01,-.410551306E+00, .532760526E+00,
     *-.151227645E-02, .182345800E-01, .358417761E-01,-.103889316E-03,
     * .395514004E+00, .100299786E-02, .138275245E-03, .288046807E-01,
     *-.127951613E-05,-.177797800E-04, .239511803E-02,-.284121147E-03,
     * .939796129E-04,-.101830861E-04, .504629929E-03, .105982946E+02,
     * .265464860E+01,-.157855689E+01,-.548140707E+01,-.181759612E-01,
     * .653535097E-01, .405331254E+00,-.726064092E-02,-.554702622E+01,
     *-.652021402E-02, .802389538E-01, .167926792E+00,-.384118806E-02,
     * .872021714E-02, .474604567E-01, .772720393E-01, .144274860E-02,
     *-.179837707E-01, .871619151E-01,
     * 0.0/
      XN=XX(1)
      YN=XX(2)
      ZN=XX(3)
      R2=XN*XN+YN*YN+ZN*ZN
      R=SQRT(R2)
```

```
      DO 1 I=1,5
      X(I)=XN
      Y(I)=YN
      Z(I)=ZN
      XN=XN*XX(1)
      YN=YN*XX(2)
      ZN=ZN*XX(3)
    1 CONTINUE
      EXPC=EXP(-R/5.2)
      IF(R2.GT.900)R2=900
      EXPR=EXP(-.06*R2)
      BB(1)=(A( 1)*Z( 1)+A( 2)*X( 1)*Z( 1)+A( 3)*Z( 3)+A( 4)*Y( 2)*Z( 1)
     *+A( 5)*Y( 2)*Z( 3)+A( 6)*X( 1)*Z( 3)+A( 7)*X( 1)*Y( 2)*Z( 1)+A( 8)
     **X( 2)*Z( 1)+A( 9)*X( 2)*Z( 3)+A(10)*X( 2)*Y( 2)*Z( 1)+A(11)*X( 3)
     **Z( 1)+A(12)*Z( 5)+A(13)*Y( 4)*Z( 1)+A(14)*X( 4)*Z( 1))*EXPC
     *+(0.0 +A(15)
     **Z( 1)+A(16)*X( 1)*Z( 1)+A(17)*Z( 3)+A(18)*Y( 2)*Z( 1)+A(19)*Y( 2)
     **Z( 3)+A(20)*X( 1)*Z( 3)+A(21)*X( 1)*Y( 2)*Z( 1)+A(22)*X( 2)*Z( 1)
     *+A(23)*X( 2)*Z( 3)+A(24)*X( 2)*Y( 2)*Z( 1)+A(25)*X( 3)*Z( 1)+A(26)
     **Z( 5)+A(27)*Y( 4)*Z( 1)+A(28)*X( 4)*Z( 1))*EXPR
      BB(2)=(B( 1)*Y( 1)*Z( 1)+B( 2)*X( 1)*Y( 1)*Z( 1)+B( 3)*Y( 1)*Z( 3)
     *+B( 4)*Y( 3)*Z( 1)+B( 5)*X( 1)*Y( 1)*Z( 3)+B( 6)*X( 1)*Y( 3)*Z( 1)
     *+B( 7)*X( 2)*Y( 1)*Z( 1)+B( 8)*X( 3)*Y( 1)*Z( 1))*EXPC
     *+(0.0 +B( 9)*Y( 1)
     **Z( 1)+B(10)*X( 1)*Y( 1)*Z( 1)+B(11)*Y( 1)*Z( 3)+B(12)*Y( 3)*Z( 1)
     *+B(13)*X( 1)*Y( 1)*Z( 3)+B(14)*X( 1)*Y( 3)*Z( 1)+B(15)*X( 2)*Y( 1)
     **Z( 1)+B(16)*X( 3)*Y( 1)*Z( 1))*EXPR
      BB(3)=(C( 1)+C( 2)*X( 1)+C( 3)*Z( 2)+C( 4)*Y( 2)+C( 5)*Y( 2)*Z( 2)
     *+C( 6)*X( 1)*Z( 2)+C( 7)*X( 1)*Y( 2)+C( 8)*X( 1)*Y( 2)*Z( 2)+C( 9)
     **X( 2)+C(10)*X( 2)*Z( 2)+C(11)*X( 2)*Y( 2)+C(12)*X( 3)+C(13)*X( 3)
     **Z( 2)+C(14)*X( 3)*Y( 2)+C(15)*Z( 4)+C(16)*Y( 4)+C(17)*X( 1)*Z( 4)
     *+C(18)*X( 1)*Y( 4)+C(19)*X( 4))*EXPC
     *+(0.0 +C(20)+C(21)*X( 1)+C(22)*Z( 2)
     *+C(23)*Y( 2)+C(24)*Y( 2)*Z( 2)+C(25)*X( 1)*Z( 2)+C(26)*X( 1)*Y( 2)
     *+C(27)*X( 1)*Y( 2)*Z( 2)+C(28)*X( 2)+C(29)*X( 2)*Z( 2)+C(30)*X( 2)
     **Y( 2)+C(31)*X( 3)+C(32)*X( 3)*Z( 2)+C(33)*X( 3)*Y( 2)+C(34)*Z( 4)
     *+C(35)*Y( 4)+C(36)*X( 1)*Z( 4)+C(37)*X( 1)*Y( 4)+C(38)*X( 4))*EXPR
      RETURN
      END
```

```
      FUNCTION RINGST(SOFFD,DST)
C
C  THIS FUNCTION CALCULATES THE STRENGTH OF THE RING CURRENT FROM
C  THE STANDOFF DISTANCE AND THE DST.
C
C  THIS FUNCTION CAN BE USED TO CALCULATE A VALUE FOR SRING
C  ONE OF THE REQUIRED PARAMETERS FOR CALCUATING THE DYNAMIC
C  MAGNETIC FIELD
C
C  IT CALCULATES THE CONTRIBUTION OF THE MAGNETOPAUSE CURRENTS TO
C  GROUND BASED SIGNATURE AND SUBTRACTS THAT COMPONENT FROM THE
C  OBSERVED VALUE OF DST.  IT ATTRIBUTES THE REMAINDER TO THE RING
C  CURRENT
C
C  INPUT PARAMETERS
C
C        SOFFD  THE STANDOFF DISTANCE OF THE MAGNETOPAUSE.  THE QUIET
C               STANDOFF DISTANCE IS 10.5 EARTH RADII.  ACCEPTABLE
C               VALUES RANGE BETWEEN 6 AND 11.  THIS VALUE IS USED TO
C               CALCULATE THE STRENGTH OF THE MAGNETOPAUSE CURRENTS AND
C               TO SCALE THE SIZE OF THE MAGNETOPAUSE.  THIS VALUE ALSO
C               SCALES THE SIZE OF THE TAIL CURRENT SYSTEM.  THE RING
C               SYSTEM IS NOT SCALED, SINCE ITS SOURCE IS PRIMARILY AT
C               RADIAL DISTANCES.
C
C        DST    DST IS THE STANDARD PUBLISHED DST VALUE IN NANOTESLA
C               THE STORMTIME DISTURBED EQUATORIAL FIELD
C
C  CON    SCALES THE EFFECT OF THE DST (ITS VALUE OF .03 IS STILL
C         SOMEWHAT UNCERTAIN)
C
      DATA CON/.03/
      SCL=10.5/SOFFD
      SCM=SCL**3
      DSTMOD=(SCM-1)*15.-DST
      RINGST=1.+DSTMOD*CON
      RETURN
      END
```

```
      FUNCTION STDOFF(VEL,DEN)
C
C  THIS FUNCTION CALCULATES THE STANOFF DISTANCE FROM THE SOLAR WIND
C  VELOCITY AND DENSITY.  IT CAN BE USED TO EVALUATE THE PARAMETER
C  SOFFD.  SOFFD IS REQUIRED FOR ALL SCALING OPERATIONS.
C
C  INPUT PARAMETERS
C
C     VEL     THE SOLAR WIND VELOCITY IN KM/SEC NEAR THE SUBSOLAR POINT
C             TYPICAL VALUES ARE 300 TO 500
C
C     DEN     THE NUMBER DENSITY OF THE SOLAR WIND IN NUMBER PER CC
C             TYPICAL VALUES ARE 5 TO 50
C
C  OUTPUT
C
C     STDOFF THE DISTANCE TO THE SUBSOLAR POINT IN RE
C
      STDOFF=98./((DEN*VEL**2)**(1./6.))
      RETURN
      END
```

Appendix D

INVARM, the B,L Code

This appendix presents a listing of subroutine INVARM, the central routine for developing the new radiation belt models. The operation and functions performed by this routine are spelled out in Section 2.3. The routine is preceded by a test routine that presents a sample of its capabilities and also provides a means for assessing its operation.

The test routine varies the distance, latitude, longitude, and universal time and asks the INVARM program to calculate the invariant for 18 different pitch angles at each location.

All input and out variables are passed via the arguments of the subroutine call. It is possible to limit the number of terms used by the internal field routine by setting NMAX to a value between 2 and 10. This must be done via labeled COMMON /GCOM/. The internal field routine checks the value of this variable. If it is set to zero as it normally is when labeled COMMON is preset by the loader or set to any value other than 2 through 10, then the internal field routine uses the maximum coefficients defined for the IGRF field. For the coefficients supplied with this program an NMAX of 11 is used. Labeled COMMON /MOMENT/ contains the dipole moment of the main field as calculated from the coefficient set that is in use. This may be used by any routine that requires it.

The calling argument for the routine are

Input variables

XLAT — The geographic latitude measured in degrees from the geographic equator, plus is north and minus is south.

XLONG — The geographic east longitude measured from Greenwich England (0-360)

R — The radial distance from the center of the earth in unit of earth radii. One radius is 6371.2 km.

YR — The year of the calculation. This variable is used by the internal magnetic field routine to calculate the epoch of the magnetic field. Whenever this value changes by 0.1 years the coefficient set for the internal field is updated. It is suggested that this value not be changed unless the drift of the internal field during the calculations is important to the analysis. The coefficients are valid from 1945 to the present. Dates earlier than 1945, will cause the routine to use the 1945 coefficient set. Use caution in predicting the field far into the future. Historically, predicting the field into the future has not been very successful.

DAY The day of the year. January 1 is DAY = 1. This is a floating point variable but it should be limited to whole numbers.

TIME The universal time in hours. This is a floating point number should represent the correct universal time to the required precision.

JSWITCH An integer variable that controls whether the external field is included in the calculation. JSWITCH negative uses the internal field only, JSWITCH = 0 or positive uses the internal plus external magnetic field.

NUMANG An integer variable that specifies the number of pitch angles for which the invariant must be calculated.

PANGLE A single variable or an array that contains the pitch angles for which the routine is to calculate the invariant. The dimension of PANGLE must be equal to or greater than NUMANG. If NUMANG is 1, then PANGLE may be a simple undimensioned variable.

Output parameters

EL A simple variable or an array dimensioned to at least NUMANG that will contain the L value for the specified location and pitch angle. If no L could be calculated EL is set to -1.0, if the mirror point is below 1.03 R_e or El is set to 100 if the field line is open or the maximum number of steps is reached by the routine.

BLOCAL The value of the magnetic field at the observation point.

BMIN The minimum value of the magnetic field along the particle line of force.

XMLONG The magnetic longitude of the minimum B point on the magnetic line of force. 0 degrees is local midnight.

XMLAT The magnetic latitude of the observation point.

BMAXAN The Mirror point magnetic field for each of the pitch angles. BMAXAN can either be a simple variable or and array dimensioned at least to order NUMANG.

XJ The value of the second invariant for each pitch angle. XJ can either be a simple variable or and array dimensioned at least to order NUMANG.

DENSTY The column density of the atmosphere along the particle's bounce path in gm/cm^2. DENSTY can either be a simple variable or and array dimensioned at least to order NUMANG.

```
      COMMON/GCOM/ ST,CT,SP,CP,AOR,BT,BP,BR,NMAX,YEARI
      DIMENSION BMAXAN(20),XJ(20),ANGLE(20),EL(20),DENSTY(20),ALPHEQ(20)
      common/temp/nlast,n2last
C
C     THIS PROGRAM PROVIDES A TEST RUN OF THE L VALUE SUBROUTINES
C
      CHARACTER*6 IAR(3)
      DATA IAR/6H INT   ,6H IN+EX,6H L AVE/
      DO 500 I=1,18
      ANGLE(I)=90-(I-1)*5

500   CONTINUE
      NUMANG=18
      IDSWIT=1
      LN=100
      YEAR=1990
      DA=1
      DO 5 IU=1,1,12
      UT=IU-1
      LN=100
      DO 4 IL=1,31,30
      FLAT=IL-1
      DO 3 ILG=1,181,180
      XLONG=ILG
      DO 2 IR=2,8,2
      R=IR-.5
      DO 1 IC=1,2
      call gettim(ihr,imin,isec,i100)
      btime=float(imin*60+isec)+float(i100)/100.
      CALL INVARM(FLAT,XLONG,R,YEAR,DA,UT,IC-2,ANGLE,NUMANG,
     *EL,BLOC,BM,XMLONG,XMLAT,BMAXAN,XJ,DENSTY)
      call gettim(jhr,jmin,jsec,j100)
      time=float(jmin*60+jsec)+float(j100)/100.-btime
      IF(IC.EQ.1)WRITE(*,103)
103   FORMAT(1H1)
      WRITE(*,101)FLAT,XLONG,R,YEAR,DA,UT,IAR(IC),BLOC,BM,XMLAT,XMLONG
101   FORMAT(//,' Lat = ',f6.1,'  Long = ',f7.1,'  R = ',f4.1,
     */,' Year = ',f7.1,'  Day = ',f5.0,'  UT = ',f6.2,'  Field =',A6,
     */,' Blocal = ',F8.5,'  Bmin = ',F8.5,'  Mlat = ',f8.3,
     *'  Mlong = ',f9.3)
      WRITE(*,102)
102   FORMAT(/,' P. Angle    B mir      2nd Inv.    L         Density',
     *'      Eq. Pitch Angle',/)
      LN=0
      DO 50 I=1,NUMANG
50    ALPHEQ(I)=ASIN(SQRT(BM/BLOC)*SIN(ANGLE(I)*.01745329))/.01745329
      write(*,100)(angle(i),bmaxan(i),xj(i),el(i),densty(i),
     *alpheq(i),i=1,numang)
10    write(*,*)nlast,n2last,time
 100  format(0Pf6.1,2f11.5,f8.3,1PE15.5,0PF13.2)
 1    CONTINUE
 2    CONTINUE
 3    CONTINUE
 4    CONTINUE
 5    CONTINUE
      END
```

```
SUBROUTINE INVARM(XLAT,XLONG,R,YR,DAY,TIME,JSWTCH,PANGLE,
     *NUMANG,EL,BLOCAL,BMIN,XMLONG,XMLAT,BMAXAN,XJ,DENSTY)
C  Version 11/95 -- Final Version
C  Written by Karl A. Pfitzer, MDSSC, 714-896-3231
C
C      PURPOSE
C         CALCULATE THE VARIOUS MAGNETIC COORDINATES OF THE PARTICLES'
C         DRIFT SHELL.  CALCULATE THE 1ST AND 2ND ADIABATIC INVARIANTS AND
C         THE L PARAMETER FOR A NUMBER OF PITCH ANGLES AT THE SPECIFIED
C         LOCATION.  ALSO DETERMINE THE LOCAL MAGNETIC FIELD, THE MAGNETIC
C         LATITUDE, THE MINIMUM MAGNETIC FIELD ON THE FIELD LINE AND THE
C         MAGNETIC LONGITUDE AT THE FIELD MINIMUM.
C
C      INPUT -- ARGUMENT LIST
C         XLAT   GEOCENTRIC GEOGRAPHIC LATITUDE IN DEGREES (+ IS NORTH)
C         XLONG  GEOCENTRIC GEOGRAPHIC LONGITUDE EAST OF GREENWHICH IN
C                DEGREES
C         R      GEOCENTRIC DISTANCE FROM THE EARTHS CENTER IN UNITS
C                EARTH RADII, RE.   RE=6371.2 KM
C         YR     THE YEAR - USED BY THE INTERNAL MAGNETIC FIELD ROUTINE
C                TO TAKE INTO ACCOUNT THE SECULAR VARIATIONS
C                (E.G. JULY 15,1964 = 1964.54)
C                NOTE*** YR SHOULD BE CHANGED ONLY EVERY FEW DAYS OR
C                MONTHS. NEW FIELD COEFFICIENTS MUST BE COMPUTED FOR
C                EVERY CHANGE IN YR, THIS COULD CAUSE A LARGE INCREASE IN
C                COMPUTER TIME. THE EARTHS FIELD CHANGES ONLY ABOUT
C                .001 GAUSS/YEAR AT THE EARTHS SURFACE.
C                IF YR IS CHANGED BY MORE THAN .1 YEAR NEW FIELD COEFFS.
C                ARE COMPUTED
C         DAY    THE DAY OF YEAR (1.-366.).  THE DAY IS USED BY THE
C                MAGNETIC FIELD ROUTINE TO CALCULATE THE TILT OF THE
C                DIPOLE AXIS FOR THE EXTERNAL FIELD ROUTINE
C                DAY MUST BE A WHOLE NUMBER AND DAY 1 IS JANUARY 1
C         TIME   UNIVERSAL TIME IN HOURS (0.000-24.0000)
C         JSWTCH A FLOW CONTROL VARIABLE
C                JSWTCH =-1 COMPUTE L USING INTERNAL FIELD ONLY
C                       = 0 COMPUTE L USING INTERNAL + EXTERNAL FIELD
C         PANGLE A SINGLE PITCH ANGLE OR AN ARRAY OF PITCH ANGLES FOR THE
C                INVARIANTS WILL BE CALCULATED.  THE PITCH ANGLE MUST BE
C                .LE. 90 AND GT.0 AND THE ARRAY MUST BE ORDERED IN
C                DESCENDING ORDER (90, 80, 70,...)
C         NUMANG THE NUMBER OF ELEMENT IN THE PANGLE ARRAY
C      OUTPUT PARAMETERS
C         EL     A SINGLE VARIABLE OR AN ARRAY OF DIMENSION NUMANG.  THIS
C                RETURNS THE L VALUE CALCLUATED FROM THE INVARIANT.
C                VARIABLE JSWTCH
C                *****NOTE****
C                SINCE THIS ROUTINE USES AN ACTUAL MAGNETOSPHERIC
C                MAGNETIC FIELD, THE FIELD LINES ARE NOT ALL CLOSED.
C                THUS L IS DEFINED ONLY IN THE INNER MAGNETOSPHERE (IN
C                THE REGION OF CLOSED DRIFT SHELLS).  AN ATTEMPT
C                TO CALCULATE L OUTSIDE OF THIS REGION WILL SET EL TO
C                100 (EL=100), SET BMIN TO THE LOCAL FIELD VALUE
C                AND SET XMLONG TO ZERO UNLESS MINIMUM B WAS PASSED PRIOR
C                TO THE DETECTION OF THE ERROR
C                IF THE MIRROR POINT FOR A GIVEN PITCH ANGLE IS BELOW 200KM
C                THEN EL IS SET TO -1
C         BLOCAL THE VALUE OF THE MAGNETIC FIELD AT THE INPUT POSITION
C                (IN GAUSS)
C         BMIN   THE MINIMUM VALUE OF B ALONG THE FIELD LINE IN GAUSS
```

```
C        XMLONG THE MAGNETIC LONGITUDE OF THE MAGNETIC FIELD MINIMUM
C               MEASURED EAST OF THE PRIME MAGNETIC MERIDIAN
C               (IN DEGREES).
C               A PRESET CONSTANT IN SUBROUTINE MGLONG ALLOWS THE USER
C               TO SELECT EITHER A CENTERED DIPOLE MAGNETIC COORDINATE
C               SYSTEM WITH ZERO AT 69 DEG W. GEOGRAPHIC, OR AN OFFSET
C               DIPOLE COORDINATE SYSTEM WITH ZERO THROUGH GREENWHICH.
C        XMLAT  THE MAGNETIC LATITUDE IN DEGREES OF THE CURRENT POSITION
C        BMAXAN A SINGLE VARIABLE OR AN ARRAY OF AT LEAST DIMENSION NUMANG
C               THAT WILL HOLD THE MIRROR POINT MAGNETIC FIELD FOR THE
C               VARIOUS PITCH ANGLE.
C        XJ     A SINGLE VARIABLE OR AN ARRAY OF AT LEAST DIMENSION NUMANG
C               THAT WILL HOLD THE VALUES OF THE SECOND INTEGRAL INVARIANT
C               FOR EACH PITCH ANGLE
C
C     CONSTANTS
C        ERR    = 0.0005  SCALES THE ERROR LIMITS FOR THE INTEGRATION
C               THE ERROR IN L IS APPROXIMATELY L*ERR
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE STEPSZ
C        SUBROUTINE BMNEXT
C        SUBROUTINE HILTEL
C        SUBROUTINE INTERP
C        SUBROUTINE INTGRT
C        SUBROUTINE INVR
C        SUBROUTINE MGLONG
C
C     VARIABLES (PARTIAL LIST TO HELP UNDERSTAND THE CODE)
C        BINTL  A REAL ARRAY THAT SAVES THE VALUE OF THE MAGNETIC FIELD
C               AT THE INPUT POSITION
C        B      A REAL ARRAY THAT HOLDS THE INSTANTANEOUS MAGNETIC FIELD
C               VECTOR AT EACH INTEGRATION STEP
C        BB     A 2 DIMENSIONED REAL ARRAY THAT HOLDS ALL OF THE
C               MAGNETIC FIELD MAGNITUDES CALCULATED AT ALL OF THE
C               INTEGRATION STEPS
C        BL     A REAL ARRAY THAT SAVES THE MAGNETIC FIELD VECTOR
C               FROM THE PREVIOUS INTEGRATION STEP
C        BMAX   THE MAGNETIC FIELD AT THE PARTICLE MIRROR POINT
C        COSINE THE COSINE OF THE GEOGRAPHIC LATITUDE
C        DAYYR  THE DAY OF THE YEAR
C        DDS    THE ESTIMATED STEP SIZE NECESSARY TO COMPLETE THE
C               INTEGRATION.  IF NO ESTIMATE IS YET POSSIBLE IT IS
C               SET TO 100.
C        DEL    SCALES THE INTEGRATION STEP SIZE.  IT IS PROPORTIONAL
C               TO THE FOURTH ROOT OF THE ERROR LIMITS.  IF IT IS
C               POSITIVE INTEGRATION WILL BE PARALLEL TO THE FIELD, IF
C               NEGATIVE IT IS ANTIPARALLEL
C        DS     THE CURRENT VALUE OF THE INTEGRATION STEP SIZE IN
C               EARTH RADII. POSITIVE IS FOR PARALLEL TO FIELD,
C               NEGATIVE FOR ANTIPARALLEL
C        IERFLG AN ERROR FLAG SET BY SUBROUTINE INTGRT.  IF NON-ZERO
C               THE INTEGRATION HAS GONE BEYOND THE SET LIMITS AND
C               MUST BE TERMINATED
C        JSW    A FLOW CONTROL PARAMETER USED BY THE MAGNETIC FIELD
C               SUBROUTINE
C        KODE   SET EQUAL TO ONE TO INDICATE TO SUBROUTINE BMNEXT
C               THAT CARESIAN COORDINATES ARE TO BE USED
C        KS     A VARIABLE TRANSMITTED TO SUBROUTINE INTERP.  IT IS
C               USED TO DETERMINE WHICH SOLUTION APPLIES TO THE
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C                PARTICULAR INTERPOLATION
C        MINFLG  INITIALLY SET TO ZERO.  IT IS SET TO ONE WHEN THE FIELD
C                MINIMUM HAS BEEN PASSED
C        N       THE CURRENT INTEGRATION STEP NUMBER
C        PICON   PI / 180.
C        Q, QL   REAL ARRAYS CONTAINING THE CURRENT AND PREVIOUS ERROR
C                ESTIMATES.  USED BY GILLS METHOD INTEGRATION ROUTINE TO
C                CONTROL ROUND OFF ERROR
C        RMIN    THE VECTOR POSITION TO THE MAGNETIC MINNIMUM
C        RMAG    THE MAGNITUDE OF THE DISTANCE TO BMIN
C        SER     ERROR CONTROL VARIABLE.  THE INTEGRATION STOPS IF
C                THE CURRENT POSITION POINT IS WITHIN DISTANCE SER OF
C                BMAX
C        SF      OUTPUT OF THE INTERPOLATION SUBROUTINE INTERP.  IT
C                INDICATES THE SCALAR DISTANCE ALONG THE FIELD WHERE
C                B IS EQUAL TO BMAX
C        SXJ     A REAL ARRAY WHICH SAVES THE INTEGRATION STEP VALUES
C                OF THE SECOND ADIABATIC INVARIANT
C        UT      UNIVERSAL TIME
C        XDS     TEMORARY VALUE USED FOR OBTAINING DISTANCE TO COMPLETION
C                OF INTEGRATION
C        XJ      FINAL VALUE OF THE SECOND ADIABATIC INVARIANT
C        XL      A REAL ARRAY HOLDING THE PREVIOUS VALUE OF THE POSITION
C                VECTOR
C        XSV     A 3 DIMENSIONED REAL ARRAY HOLDING ALL OF THE POSITION
C                VECTORS ALONG THE INTEGRATION PATH
C        XXJ     INTERPOLATED VALUE OF THE SECOND ADIABATIC INVARIANT
C        YEAR    THE YEAR
C        ZP      THE Z COMPONENT OF THE POSITION VECTOR IN CENTERED
C                DIPOLE COORDINATES
C
C     VERSION 6/91
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER AT MCDONNELL
C     DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON BEACH CALIF.
C     PHONE (714) 896-3231.
C
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      common/temp/nlast,n2last
      DIMENSION BB(100,4),BB2(100,4),B(3),B2(3),X(3),X2(3),S(100),
     *S2(100),DEN(100),DEN2(100)
      DIMENSION EL(*),PANGLE(*),BMAXAN(*),XJ(*),BLL(3),BLL2(3)
      DIMENSION XX(3),BINTL(3),DENSTY(*)
      DATA PICON/.01745329252/
      DATA ERR/.0005/
      DATA CONI/.95/
C
C     OBTAIN THE CARTESIAN COMPONENTS OF THE POSITION VECTOR
C
C     CHECK THE PITCH ANGLES, THEY MUST BE BETWEEN 90 AND 0 AND THEY MUST
C     BE IN DESCENDING ORDER (IE. 90,85,80,....
C
      NMANG=NUMANG
      BMIN=100
      CALL CHECK(PANGLE,NMANG,IER)
      IF (IER.GT.0) THEN
      WRITE(*,*)'Pitch angle error, must be monotonic and >0 & <=90'
      DO 5 I=1,NMANG
```

```
            XJ(I)=-1
            BMAXAN(I)=-1
            EL(I)=-1
            DENSTY(I)=-1
 5          CONTINUE
          RETURN
          ENDIF
C
          COSINE=COS(XLAT*PICON)
          XX(1)=R*COSINE*COS(XLONG*PICON)
          XX(2)=R*COSINE*SIN(XLONG*PICON)
          XX(3)=R*SIN(XLAT*PICON)
C
C         ROTATE TO DIPOLE COORDINATES (FIRST ROTATE ABOUT Z 291 DEGREES
C         THEN ABOUT THE NEW Y 11.7 DEGREES TO THE DIPOLE AXIS)
C
          ZP=(XX(1)*.3583679495-XX(2)*.9335804265)*.2027872954
         *+XX(3)*.9792228106
C
C         EVALUATE THE MAGNETIC LATITUDE
          XMLAT=90.-ACOS(ZP/R)/PICON
C
C         SET THE MAGNETIC LONGITUDE TO ZERO.  IF MINIMUM B IS REACHED
C         PRIOR TO AN ERROR BEING DETECTED XMLONG IS UPDATED TO REFLECT
C         MAGNETIC LONGITUDE AT MINIMUM B
          XMLONG=0.
C
C         SET UP THE COMMON BLOCK INPUT VARIABLES FOR THE MAGNETIC FIELD
C         SUBROUTINE
          YEAR=YR
          UT=TIME
          DAYYR=DAY
          JSW=JSWTCH
          KODE=1
          IBEFLG=0
C
C         EVALUATE THE MAGNETIC FIELD AT THE STARTING POINT
          CALL BMNEXT(XX,B,BB(2,1))
          BLOCAL = BB(2,1)
          BB2(2,1)=BB(2,1)
C
C         SAVE THE INITIAL POSITION AND MAGNETIC FIELD VECTORS
          DO 10 I=1,3
          BINTL(I)=B(I)
          B2(I)=B(I)
          XL(I)=XX(I)
          X2(I)=XX(I)
          XSV(2,I,1)=XX(I)
 10       CONTINUE
          RSV(2)=R
C
C         EXIT THE ROUTINE IF POSITION IS OVER THE POLAR CAP OR DISTANCE
C         IS TOO LARGE OR MAGNETIC FIELD IS TOO WEAK
          IF(ABS(XMLAT).GT.75..OR.R.GT.12..OR.BB(2,1).LT..00025) THEN
          NMANG=0
          BMAXAN(1)=100.
          GOTO 218
          ENDIF
C
C         SET UP THE INITIAL VALUES FOR THE VARIABLES
```

```
      NLAST=2
      N2LAST=2
      S(2)=0.
      S2(2)=0.
      DEN(2)=0
      DEN2(2)=0
      DDS=100.
      MINFLG=0
C
C     SET BMIN TO LOCAL FIELD VALUE.  IF MINIMUM B IS REACHED PRIOR
C     TO ERROR DETECTION BMIN IS UPDATED TO MINIMUM B.
      BMIN=BB(2,1)
C
C     SET UP THE ERROR LIMITS FOR THE INTEGRATION
      SER=SQRT(ERR)
C     STEP SIZE GOES AS ERROR TO THE .25 POWER
      DEL=-2.5*ERR**.25
      DS=SER
C
C     STEP ONCE IN THE INCREASING FIELD DIRECTION AND SET STEP
C     PARAMETERS TO INTEGRATE IN THE DECREASING FIELD DRECTION
      IFLAG=0
      IF(XMLAT.GT.0.)GO TO 30
 20   DEL=-DEL
      DS=-DS
 30   N=2
      DO 31 I=1,3
      Q(I)=0
      X(I)=XL(I)
 31   CONTINUE
      CALL INTGRT(X,B,BB,S,DEN)
      IFLAG=IFLAG+1
      IF((BB(3,1).LT.BB(2,1)).AND.(IFLAG.LE.1)) GOTO 20
      S(1)=S(3)
      DEN(1)=DEN(3)
      BB(1,1)=BB(3,1)
      DO 34 I=1,3
      XL(I)=X(I)
      BL(I)=B(I)
      xsv(1,i,1)=xsv(3,i,1)
      Q(I)=0
      X(I)=XX(I)
      B(I)=BINTL(I)
 34   CONTINUE
      RSV(1)=SQRT(XL(1)**2+XL(2)**2+XL(3)**2)
      DELSV=DEL
C
C     BEGIN THE FIELD LINE INTEGRATION.  THE INTEGRATION USES A VARIABLE
C     STEPSIZE WHICH IS DEPENDENT ON THE CURVATURE OF THE FIELD LINE
C     AND ON THE DISTANCE EACH POINT IS FROM EARTH CENTER (A MEASURE
C     OF THE MAGNETIC FIELD STRENGTH).  THE INITIAL INTEGRATION IS A
C     LINE INTEGRAL OF THE MAGNETIC FIELD UNIT VECTOR.  THIS INTEGRATION
C     LOOP ALSO SAVES ALL OF THE VARIABLES WHICH ARE LATER NEEDED TO
C     EVALUATE THE SECOND INTEGRAL INVARIANT.
C
      DO 216 IA=1,NMANG
      DEL=DELSV
      DDS=100
      BMAX=BB(2,1)/SIN(PANGLE(IA)*PICON)**2
      N=NLAST
```

```
      IF (IA.NE.1)THEN
      DO 41 I=1,3
        BL(I)=BLL(I)
        Q(I)=0
 41     CONTINUE
      DS=DSL
      IF(N.GE.3)THEN
      CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,KS)
      IF(ABS(SF).GT.ABS(S(N))) THEN
        XDS=SF-S(N)
        IF(ABS(XDS).LE.SER)GOTO 100
        DDS=CONI*XDS
      ENDIF
      ENDIF
      ENDIF
C
 40   CALL STEPSZ(X,B,BB)
 45   CALL INTGRT(X,B,BB,S,DEN)
C
C     IF FIELD IS STILL DECREASING RELOOP
      IF(BB(N,1).LT.BB(N-1,1)) GO TO 40
C
C     IF MINIMUM VALUES HAVE BEEN CALCULATED, JUMP OVER MINIMUM ROUTINES
C     WHEN THE CURRENT VALUE OF B EXCEEDS THE LAST, FIND THE
C     INTERPOLATED MINIMUM MAGNETIC FIELD VALUE AND USE THIS VALUE TO
C     UPDATE THE VALUE OF BMAX TO REFLECT THE AVERAGE DRIFT SHELL (IF
C     AVERAGE SHELLS ARE REQUIRED)
C     USE THE DISTANCE, SF, TO THE FIELD MINIMUM TO DETERMINE THE
C     MAGNETIC LONGITUDE OF THE FIELD MINIMUM
C
      IF(MINFLG.NE.0) GO TO 50
      print 999,bb(n-2,1),bb(n-1,1),bb(n,1),s(n-2),s(n-1),s(n)
999   format(6e12.5)
      CALL INTERP(BB(N-2,1),S(N-2),BMIN,SF,-1)
      CALL MGLONG(XSV(N-2,1,1),S(N-2),SF,XMLONG,RMIN(1),RMAG)
      MINFLG=1
C
C     CONTINUE STEPPING ALONG THE FIELD LINE AS LONG AS B IS LESS THAN
C     BMAX AND THE INTEGRATION IS MORE THAN A DISTANCE SER FROM BMAX
C     IF BMAX HAS BEEN EXCEEDED, EXIT TO INTERPOLATION SCHEME
C
 50   IF(BB(N,1).GE.BMAX) GO TO 70
C     IF WE ARE OUTSIDE OF VALID REGION EXIT
      IF(IERFLG.NE.0)THEN
      NMANG=IA-1
      IF(IERFLG.GT.0)THEN
        BMAXAN(IA)=100
      ELSE
        BMAXAN(IA)=-1
      ENDIF
      GOTO 218
      ENDIF

      CALL INTERP (BB(N-2,1),S(N-2),BMAX,SF,3)
      DDS=100.
C
C     IF S DOES NOT INCREASE MONOTONICALLY, IGNORE INTERPOLATION
C     AND RELOOP
      IF(ABS(SF).LE.ABS(S(N))) GO TO 40
      XDS=SF-S(N)
```

```
C
C     IF WITHIN SER OF BMAX STOP INTEGRATION GO GET VALUE OF INVARIANT
      IF(ABS(XDS).LT.SER) GO TO 100
      DDS=CONI*XDS
C     RELOOP
      GO TO 40
C
C
C     THE FUNCTION SQRT(1-B/BMAX) DOES NOT EXIST FOR B GREATER THAN BMAX
C     IF PREVIOUS STEP IS NOT WITHIN SER OF BMAX INTERPOLATE TO FIND
C     A STEP SIZE THAT WILL GET CLOSE TO BUT NOT EXCEED BMAX
C
 70   CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,3)
C
      IF(ABS(SF-S(N)).LT.SER) THEN
      CALL INTERP(BB(N-2,1),RSV(N-2),BMAX,RS,3)
      IF (RS.LT.1.02)THEN
        NMANG=IA-1
        BMAXAN(IA)=-1
        GOTO 218
      ENDIF
      ENDIF

C     SET UP THE STEP SIZE AND RESET INTEGRATION VALUES TO THE PREVIOUS
C     STEP
C
      N=N-1
      XDS=DS
      DO 80 I=1,3
      X(I)=XL(I)
      Q(I)=QL(I)
      B(I)=BL(I)
 80   CONTINUE
      IF(ABS(SF).GT.ABS(S(N))) XDS=0.9*(SF-S(N))
C     IF THE STEP SIZE IS LESS THAN SER, THE PREVIOUS STEP IS CLOSE
C     ENOUGH EXIT TO INVARIANT CALCULATION
      IF(ABS(XDS).LT.SER) GOTO 100
      IF(ABS(XDS).GE.ABS(DS))THEN
      DS=DS/2
      ELSE
      DS=XDS
      ENDIF
 85   CALL INTGRT(X,B,BB,S,DEN)
C
C     IF LAST STEP IS STILL PAST BMAX TRY THE INTERPOLATION SCHEME AGAIN
 90   IF(BB(N,1).GT.BMAX) GO TO 70
C
C     INTERPOLATE TO SEE IF THE INTERPOLATION STEP IS CLOSE ENOUGH
C     TO BMAX. IF IT IS NOT, INTERPOLATE AGAIN AND TRY TO COME CLOSER
C
      CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,3)
C
C     IF WE ARE CLOSE ENOUGH EXIT THE INTEGRATION LOOP
      IF(ABS(SF-S(N)).LT.SER) THEN
      CALL INTERP(BB(N-2,1),RSV(N-2),BMAX,RS,3)
      IF (RS.LT.1.02)THEN
        NMANG=IA-1
        BMAXAN(IA)=-1
        GOTO 218
      ELSE
```

```
        GOTO 100
      ENDIF
      ENDIF

      DS=DS/2
      IF(ABS(SF).GT.ABS(S(N))) DS=CONI*(SF-S(N))
      CALL INTGRT(X,B,BB,S,DEN)
      GO TO 90
C
C     THE FIELD MAXIMUM HAS NOW BEEN REACHED.  THE STORED VALUES
C     OF THE MAGNETIC FIELD AND THE PATH LENGTH VALUES CAN NOW BE
C     USED TO EVALUATE THE SECOND INVARIANT.
C
 100  DSL=DS
      IF(N.LT.3) THEN
      XJ(IA)=0
      BMAXAN(IA)=BMAX
      DO 108 I=1,3
 108    BLL(I)=BL(I)
      GOTO 216
      ELSEIF(N.EQ.3)THEN
      DS=.5*(S(N-1)-S(N))
      CALL INTGRT(X,B,BB,S,DEN)
      KS=2
      ELSE
      KS=3
      ENDIF
      NLAST=N
      DO 109 I=1,3
      BLL(I)=BL(I)
109   CONTINUE
      DSL=DS
C
C     CALL THE ROUTINE WHICH DETERMINES THE SECOND INVARIANT FROM
C     FROM THE STORED VALUES
C
 110  CALL INVR(BMAX,BB,S)
C
C     INTERPOLATE TO GET THE BEST FIT
C
      CALL INTERP(BB(N-2,1),SXJ(N-2),BMAX,XXJ,KS)
      CALL INTERP(BB(N-2,1),DEN(N-2),BMAX,XDN,KS)
C
C     SAVE THE  VALUES OF THE FIRST AND SECOND INVARIANT
      XJ(IA)=ABS(XXJ)
      DENSTY(IA)=ABS(XDN)
      BMAXAN(IA)=BMAX
C
C
C     THE INTEGRAL HAS NOW BEEN EVALUATED FROM THE STARTING POINT
C     THROUGH THE MINIMUM VALUE OF B TO BMAX.
C
C     WE MUST INTEGRATE THE REST OF THE LINE --- TURN THE STARTING
C     POINTS AROUND AND RESET THE INITIAL VALUES AND INTEGRATE TO THE
C     OTHER BMAX
C
      DEL=-DELSV
      BB2(1,1)=BB(3,1)
      SXJ(1)=SXJ(3)
      S2(1)=S(3)
```

```
      DEN2(1)=DEN(3)
      DS=S(2)-S(3)
      IF(IA.EQ.1.or.N2LAST.LE.2)THEN
      N=2
      ELSE
      N=N2LAST
      DO 117 I=1,3
        BL(I)=BLL2(I)
117     CONTINUE
      DS=S2(N)-S2(N-1)
      if(n.le.2)write(*,*)' bad n'
      CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
      IF(ABS(SF).GT.ABS(S2(N))) THEN
        XDS=SF-S2(N)
        IF(ABS(XDS).LE.SER)GOTO 200
        DDS=CONI*XDS
      ENDIF
      ENDIF
      CALL STEPSZ(X2,B2,BB2)

      IF(ABS(BB(2,1)-BMAX)/BMAX.LT.ERR.OR.IBEFLG.NE.0) GO TO 216
      CALL INTERP(BB(2,1),S(2),BMAX,SF,1)
      IF(ABS(SF).LT.SER) THEN
      CALL INTERP(BB(2,1),SXJ(2),BMAX,XXJ,1)
      GOTO 215
      ENDIF
C
      IF(ABS(SF).LT.ABS(DS))DS=.7*SF
      DO 120 I=1,3
      Q(I)=0.
 120  CONTINUE
      N=N2LAST

      GO TO 140
C
C     BEGIN INTEGRATING THE SECOND PART
 130  CALL STEPSZ(X2,B2,BB2)
 140  CONTINUE
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
      DDS=100.
C
C     STOP INTEGRATION IF BMAX HAS BEEN PASSED
      IF(BB2(N,1).GE.BMAX) GO TO 150
      IF(IERFLG.NE.0)THEN
      IF(IERFLG.GT.0)THEN
        BMAXAN(IA)=100
      ELSE
        BMAXAN(IA)=-1
      ENDIF
       NMANG=IA-1
       GOTO 218
      ENDIF

      CALL INTERP (BB2(N-2,1),S2(N-2),BMAX,SF,3)
C
C     IGNORE INTERPOLATION IF RESULT IS NOT MONOTONIC
      DDS=100
      IF(ABS(SF).LE.ABS(S2(N))) GOTO 130

      XDS=SF-S2(N)
```

```
C
C     STOP INTEGRATION IF WITHIN SER OF BMAX
      IF(ABS(XDS).LT.SER) GO TO 200
      DDS=CONI*XDS
      GO TO 130
C
C     BMAX HAS BEEN PASSED, BEGIN INTERPOLATION SCHEME TO FIND A POINT
C     CLOSE TO BMAX BUT LESS THAN IT.
C
 150  CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
      IF(ABS(SF-S2(N)).LE.SER) THEN
      CALL INTERP(BB2(N-2,1),RSV(N-2),BMAX,RS,3)
      IF (RS.LT.1.02)THEN
         NMANG=IA-1
         BMAXAN(IA)=-1
         GOTO 218
      ENDIF
      ENDIF

      N=N-1
      XDS=DS
      IF(ABS(SF).GT.ABS(S2(N))) XDS=0.9*(SF-S2(N))
      IF(ABS(XDS).LT.SER) GOTO 200
      IF(ABS(XDS).GE.ABS(DS))THEN
      DS=DS/2
      ELSE
      DS=XDS
      ENDIF
      DO 160 I=1,3
      X2(I)=XL(I)
      Q(I)=QL(I)
      B2(I)=BL(I)
 160  CONTINUE
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
 170  IF(BB2(N,1).GT.BMAX) GO TO 150
C
C     IF THE POINT IS LESS THAN BMAX MAKE SURE IT IS CLOSE ENOUGH. IF
C     NOT, TRY TO GET CLOSER
C
      CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
      IF(ABS(SF-S2(N)).LT.SER) THEN
      CALL INTERP(BB2(N-2,1),RSV(N-2),BMAX,RS,3)
      IF (RS.LT.1.02)THEN
        NMANG=IA-1
        BMAXAN(IA)=-1
        GOTO 218
      ELSE
        GOTO 200
      ENDIF
      ENDIF
      DS=DS/2
      IF(ABS(SF).GT.ABS(S2(N))) DS=CONI*(SF-S2(N))
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
      GO TO 170
C
C     INTEGRAL IS COMPLETE USE STORED VALUES TO GET INVARIANT
 200  CALL INVR(BMAX,BB2,S2)

      CALL INTERP(BB2(N-2,1),SXJ(N-2),BMAX,XXJ,3)
      CALL INTERP(BB2(N-2,1),DEN2(N-2),BMAX,XDN,3)
```

```
      N2LAST=N
      DO 205 I=1,3
      BLL2(I)=BL(I)
 205  CONTINUE
C
C     ADD IN REMAINING CONTRIBUTION OF SECOND INVARIANT
 215  XJ(IA)=XJ(IA)+ABS(XXJ)
      DENSTY(IA)=DENSTY(IA)+ABS(XDN)
C
 216  CONTINUE
C
C
C     WE ARE DONE WITH ALL THE INTEGRALS - SET UP ANY ERROR VALUES
C
218   IF(NMANG.LT.NUMANG) THEN
      DO 219 I=NMANG+1,NUMANG
        XJ(I)=BMAXAN(NMANG+1)
        BMAXAN(I)=BMAXAN(NMANG+1)
        DENSTY(I)=BMAXAN(NMANG+1)
219     CONTINUE
      ENDIF
C
C     IF INVARIANT EXIST CALCULATE L'S
C
      DO 220 I=1,NUMANG
      IF(BMAXAN(I).GT.0.AND.BMAXAN(I).NE.100)THEN
        CALL HILTEL(BMAXAN(I),XJ(I),EL(I))
      ELSEIF(BMAXAN(I).LT.0) THEN
          EL(I)=-1
      ELSE
          EL(I)=100
      ENDIF
220   CONTINUE
      RETURN
      END
```

```
 SUBROUTINE CHECK(PANGLE,NUMANG,IER)
      DIMENSION PANGLE(*)
C       CHECK TO SEE IF THE PITCH ANGLES ARE BETWEEN 0 AND 90 AND THE THE
C       PITCH ANGLE ARRAY IS MONOTONICALLY DECREASING
      IER=0
      IF (PANGLE(1).GT.90..OR.PANGLE(1).LE.0) IER=1
      IF (NUMANG.GT.1) THEN
       DO 10 I=2,NUMANG
       IF(PANGLE(I).GT.PANGLE(I-1)) IER=1
       IF(PANGLE(I).GT.90..OR.PANGLE(I).LE.0) IER=1
 10      CONTINUE
      ENDIF
      RETURN
      END
```

```
 FUNCTION ADENS(X)
C
C     DETERMINE THE AVERAGE ATMOSPHERIC DENSITY AT A GIVEN ALTITUDE IN
C     GM/(CM2*Re)/3
C
      DIMENSION X(3)
C     NOMINAL VALUE OF F10.7
      DATA F107/114./
C     CONSTANT THAT CONVERTS TO CENTIMETERS AND APPLIES THE DIVIDE BY
C     THREE FROM GILL'S METHOD
C     5.7339E-3=6.371E8*2.7E-11/3
      R=SQRT(X(1)**2+X(2)**2+X(3)**2)
      IF (R.GT.3) THEN
      ADENS=0
      ELSE
      A=0.99+.518*SQRT(F107/55)
      R=(R-1)*6371
      IF (R.LT.110) THEN
        ADENS=0
      ELSE
        CON=(120-R)/(A*SQRT(R-103))
        ADENS=5.7339E-3*EXP(CON)
      ENDIF
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE INVR(BMAX,BB,S)
C
C     PURPOSE
C        TO CALCULATE THE VALUE OF THE SECOND INVARIANT
C
C     METHOD
C        USE THE VALUES STORED IN THE S AND BB ARRAYS TO EVALUATE THE
C        INTEGRAL  SQRT(1-BB/BMAX) ALONG THE FIELD LINE.  USE THE
C        SAME INTERGRATION METHOD (GILLS METHOD) USED IN INTEGRATING
C        THE FIELD LINE
C
C     INPUT -- COMMON BLOCK INTPAR
C        N        THE NUMBER OF INTEGRATION STEPS
C        BMAX     THE VALUE OF THE MAXIMUM MAGNETIC FIELD (THE POINT
C                 WHERE THE PARTICLE HAS ITS MIRROR POINT)
C        BB       A REAL 2 DIMENSIONED ARRAY CONTAINING ALL OF THE
C                 MAGNETIC FIELD MAGNITUDES CALCULATED IN THE FIELD LINE
C                 INTEGRATION
C        S        AN ARRAY THAT HOLDS THE TOTAL INTEGRATED PATH LENGHT ALONG
C                 FIELD LINE
C     OUTPUT -- COMMON BLOCK INTPAR
C        SXJ      THE VALUES OF THE SECOND INVARIANT INTEGRATION AT
C                 EACH INTEGRATION STEP.  SXJ(N) CONTAINS THE BEST
C                 APPROXIMATION TO THE VALUE OF THE SECOND INVARIANT.
C                 THE SAVING OF THE STEPS PERMITS THE USE OF INTERPOLATION
C                 SCHEMES TO OBTAIN A MORE ACCURATE VALUE OF THE INVARIANT
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     CONSTANTS
C        P29      1.0-SQRT(.5)
C        OP7      1.0+SQRT(.5)
C
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      DIMENSION BB(100,4),S(100)

      DIMENSION CON(4)
      DATA (CON(I),I=1,4)/.5,.29289322,1.70710678,.5/
      SXJ(2)=0.
C
C     START THE INTEGRATION LOOP.
C     THIS IS GILLS METHOD MADE SIMPLE IF ALL THE POINTS ARE GIVEN
C     CUMULATIVE ROUND OFF ERROR CONTROL IS NOT IMPLEMENTED
      NN=N-1
      DO 210 K=2,NN
      TEMP1=0
      DO 110 I=1,4
        IF(BB(K,I).GE.BMAX) GO TO 110
        ROOT=SQRT(1.-BB(K,I)/BMAX)
        TEMP1=TEMP1+CON(I)*ROOT
 110    CONTINUE
      DELS=(S(K+1)-S(K))/3.
      SXJ(K+1)=SXJ(K)+TEMP1*DELS
 210  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STEPSZ(X,B,BB)
C
C     PURPOSE
C        DETERMINE THE STEP SIZE FOR THE NEXT INTEGRATRION STEP
C
C     METHOD
C        THE STEP SIZE OF THE RUNGE KUTTA INTEGRATION IS A FUNCTION
C        OF THE ERROR LIMITS, THE CURVATURE OF THE FIELD LINE, THE
C        GRADIENT IN THE MAGNETIC FIELD, AND THE ESTIMATED DISTANCE
C        TO THE END OF THE INTEGRATION.
C
C     INPUT -- COMMON BLOCK INTPAR
C        DEL     A PARAMETER SET UP BY THE CALLING PROGRAM TO SCALE THE
C                STEP SIZE.  IT DEPENDS ON THE ERROR LIMITS OF THE
C                INTEGRATION.
C        B       A REAL ARRAY WHICH CONTAINS THE MAGNETIC FIELD VECTOR
C                AT THE CURRENT STEP
C        BL      A REAL ARRAY WHICH CONTAINS THE MAGNETIC FIELD VECTOR
C                AT THE PREVIOUS STEP
C        BB      A 2 DIMENSIONED REAL ARRAY
C                BB(N,1) IS THE MAGNETIC FIELD MAGNITUDE AT THE CURRENT
C                STEP
C                BB(N-1,1) IS THE MAGNETIC FIELD MAGNITUDE AT THE
C                PREVIOUS STEP
C        DDS     THE ESTIMATED STEP SIZE REQUIRED TO COMPLETE THE
C                INTEGRATION
C
C     INPUT/OUTPUT -- COMMON BLOCK INTPAR
C        DS      ON ENTRY TO THE ROUTINE DS CONTAINS THE SIZE OF THE
C                LAST STEP.  THE ROUTINE RESETS THE VALUE TO THE BEST
C                STEP SIZE FOR THE NEXT INTEGRATION STEP.
C
C     CALLING SUBROUTINES
C        INVARM
C
C     TEMPORARY VARIABLES
C        CURVMN THE MINIMUM ACCEPABLE CURVATURE.  THIS LIMITS THE STEP
C                SIZE IN THE VICINITY OF THE EARTH WHERE THE FIELD
C                CHANGES RAPIDLY
C        CURV    THE CURVATURE OF THE FIELD LINE
C
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      DIMENSION BB(100,4),B(3),X(3)

C
C     DETERMINE THE MINIMUM CURVATURE
C
      CURVMN=1.6667/(X(1)**2+X(2)**2+X(3)**2)**(.75)
C
C     DETERMINE THE CURVATURE OF THE FIELD BY USING THE RATE OF CHANGE
C     OF THE UNIT FIELD VECTOR OVER THE LAST STEP
C
      CURV=0.
      DO 50 I=1,3
      CURV=CURV+((B(I)/BB(N,1)-BL(I)/BB(N-1,1))/DS)**2
   50 CONTINUE
      CURV=SQRT(CURV)
      CURV=AMAX1(CURV,CURVMN)
```

```
C
C     SET UP THE NEW STEP SIZE AND LIMIT THE STEP SIZE TO LESS THAN 2.8
C     EARTH RADDII TO PREVENT THE INTEGRATION FROM STEPPING OUT OF THE
C     VALID FIELD REGION
C
      DS=DEL/CURV
      DS=SIGN(AMIN1(ABS(DS),1.0),DS)
      IF(N.LE.3) DS=DS*(N*2-3)*.2
C
C     IF THE DISTANCE TO THE END OF THE INTEGRATION IS SMALLER THAN THE
C     NEW STEP SIZE, SET THE STEP SIZE TO THE SMALLER VALUE.
C
      IF(ABS(DDS).LT.ABS(DS)) DS=DDS
      RETURN
      END
```

```
      SUBROUTINE INTGRT(X,B,BB,S,DEN)
C
C     PURPOSE
C        THIS SUB MODULE PERFORMS A SINGLE RUNGE-KUTTA INTEGRATION
C        STEP AND UPDATES ALL OF THE VARIABLES IN THE INTEGRATION LOOP
C
C     METHOD
C        PERFORM A SINGLE FOURTH ORDER INTEGRATION STEP USING GILLS
C        METHOD OF INTEGRATION (REF. S. GILL CAMBRIDGE PHILOSOPHICAL
C        SOCIETY PROCEEDINGS VOL. 47, 1951)
C
C     INPUT -- COMMON BLOCK INTPAR
C        DS      THE INTEGRATION STEP SIZE IN UNITS OF EARTH RADII.
C                THE INTEGRATION MOVES THE SPACE COORDINATE A DISTANCE
C                DS ALONG THE MAGNETIC FIELD LINE. IF DS IS POSITIVE,
C                MOTION IS IN THE DIRECTION OF THE FIELD. IF DS IS
C                NEGATIVE MOTION IS ANTI-PARALLEL TO THE FIELD.
C
C     INPUT/OUTPUT -- COMMON BLOCK INTPAR
C        N       THE INTEGRATION STEP NUMBER. IT IS INCREMENTED BY
C                ONE AT THE END OF THIS ROUTINE. (NOTE N=2 IS THE
C                BEGINNING OF THE INTEGRATION)
C        X       A REAL ARRAY GIVING THE VECTOR LOCATION OF THE
C                INTEGRATION VARIABLE.
C                INPUT - THE INITIAL POSITION PRIOR TO THE INTEGRATION
C                        STEP
C                OUTPUT- THE FINAL VALUE AFTER THE INTEGRATION STEP
C        B       A REAL ARRAY CONTAINING THE VECTOR MAGNETIC FIELD
C                IN GAUSS
C                INPUT - THE VECTOR FIELD BEFOR THE INTEGRATION STEP
C                OUTPUT- THE VECTOR FIELD AFTER THE STEP
C        Q       A REAL ARRAY CONTAINING AN ERROR CONTROL VARIABLE
C                USED BY GILLS INTEGRATION METHOD
C                INPUT - ERROR FROM PREVIOUS STEP
C                OUTPUT- ERROR AFTER PRESENT STEP FOR INPUT TO SUBSEQUENT
C                        STEPS
C
C     OUTPUT -- COMMON BLOCK INPTAR
C        S       A REAL ARRAY WHICH SAVES EACH OF THE DISTANCES (SINCE
C                THE START OF THE INTEGRATION) ALONG THE MAGNETIC FIELD
C                LINE.
C                S(2)=0
C                S(N+1)=S(N)+DS ETC.
C        XSV     A REAL 3 DIMENSIONED ARRAY WHICH SAVES THE VECTOR
C                POSITION IN EARTH RADII FOR EACH OF THE INTEGRATION
C                STEPS.  XSV(N,1,1), XSV(N,2,1), XSV(N,3,1) ARE VECTOR
C                CARTESIAN POSITION COORDINATES CORESPONDING TO POSITION
C                S(N) ON THE FIELD LINE
C        BB      A REAL 2 DIMENSIONED ARRAY WHCIH SAVES THE MAGNITUDE
C                OF THE MAGNETIC FIELD FORM EACH INTEGRATION STEP.
C                BB(N,1) IS MAGNETIC FIELD VALUE AT DISTANCE S(N).
C                BB(N-1,2), BB(N-1,3), BB(N-1,4) ARE THE INTERMEDIATE
C                VALUES OF THE FIELD USED BY GILLS METHOD TO GET FROM
C                BB(N-1,1) TO BB(N,1).
C        XL      A REAL ARRAY WHICH SAVES THE INITIAL POSITION VALUES
C                PRIOR TO STARTING THE INTEGRATION STEP
C        BL      A REAL ARRAY WHICH SAVES THE VECTOR MAGNETIC FIELD
C                VALUES PRIOR TO STARTING THE INTEGRATION STEP
C        QL      A REAL ARRAY WHICH SAVES THE INITIAL VALUES OF THE
C                ERROR CONTROL VARIABLE
```

```
C          IERFLG AN ERROR CONTROL INDICATOR WHICH IS USED BY THE CALLING
C                 PROGRAM TO CONTROL THE PROGRAM FLOW
C                 IERFLG = 0 NO ERROR
C                 IERFLG = 1 INTEGRATION IS OUTSIDE VALID FIELD LIMITS
C                            OR THE MAXIMUM STEP NUMBER (100) HAS BEEN
C                            REACHED.
C
C       CONSTANTS
C          P29    1.0-SQRT(0.5)
C          OP7    1.0+SQRT(0.5)
C
C       VARIABLES
C          P5DS   .5 * STEP SIZE
C          P29DS  (1.0-SQRT(0.5)) * STEP SIZE
C          OP7DS  (1.0-SQRT(0.5)) * STEP SIZE
C          RR,SS  REAL ARRAYS USED BY GILLS METHOD TO MINIMIZE COMPUTER
C                 TIME AND MINIMIZE ROUNDOFF ERROR
C
C       CALLING SUBROUTINES
C          SUBROUTINE INVARM
C
C       SUBROUTINES REQUIRED
C          SUBROUTINE BMNEXT
        COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
C
        COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
       *RSV(100),RMIN(3),RMAG,IDSW,
       *QL(3),Q(3),BL(3),SXJ(100),DDS
        DIMENSION BB(100,4),B(3),X(3),S(100),DEN(100)
        DIMENSION SS(3),RR(3)
        DATA P29,OP7/.29289322,1.70710678/
        IERFLG=0
C       SAVE THE INITIAL VALUES.  THESE INITIAL VALUES MAY BE NEEDED IF
C       IF THE INTEGRATION STEP IS UNSUCCESSFUL (GOES TOO FAR) AND THE
C       STEP MUST BE REPEATED.
C
        DO 65 I=1,3
        XL(I)=X(I)
        QL(I)=Q(I)
        BL(I)=B(I)
        Q(I)=0
 65     CONTINUE
C
C       SET UP THE CONSTANST NEEDED BY THE INTEGRATION LOOP
C
        P5DS=.5*DS
        P29DS=P29*DS
        OP7DS=OP7*DS
C
C       BEGIN GILLS METHOD (GILL 1951) OF FOURTH ORDER INTEGRATION
C
        TEMP2=P5DS*ADENS(X)
        DO 70 I=1,3
        SS(I)=P5DS*B(I)/BB(N,1)
        RR(I)=SS(I)-Q(I)
        X(I)=X(I)+RR(I)
        Q(I)=Q(I)+3.*RR(I)-SS(I)
        XSV(N,I,2)=X(I)
 70     CONTINUE
        TEMP2=TEMP2+P29DS*ADENS(X)
```

```
      CALL BMNEXT(X,B,BB(N,2))
      DO 71 I=1,3
      SS(I)=P29DS*B(I)/BB(N,2)
      RR(I)=SS(I)-P29*Q(I)
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N,I,3)=X(I)
 71   CONTINUE
      TEMP2=TEMP2+OP7DS*ADENS(X)
      CALL BMNEXT(X,B,BB(N,3))
      DO 72 I=1,3
      SS(I)=OP7DS*B(I)/BB(N,3)
      RR(I)=SS(I)-OP7*Q(I)
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N,I,4)=X(I)
 72   CONTINUE
      TEMP2=TEMP2+P5DS*ADENS(X)
      CALL BMNEXT(X,B,BB(N,4))
      DO 73 I=1,3
      SS(I)=P5DS*B(I)/BB(N,4)
      RR(I)=(SS(I)-Q(I))/3.
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N+1,I,1)=X(I)
 73   CONTINUE
      N=N+1
C
C     SAVE THE CURRENT DISTANCE ALONG THE FIELD LINE
C
      S(N)=S(N-1)+DS
      DEN(N)=DEN(N-1)+TEMP2
C
C     OBTAIN THE CURRENT VALUES OF THE MAGNETIC FIELD
C
      CALL BMNEXT(X,B,BB(N,1))
C
C     IF N IS TOO BIG, SET ERROR FLAG
      IF(N.GE.100) IERFLG=1
C
C     IF OUTSIDE INTEGRATION LIMITS SET ERROR FLAG
      R=X(1)**2+X(2)**2+X(3)**2
      RSV(N)=SQRT(R)
C     IF EXTERNAL FIELD IS USED STAY WITHIN VALID REGION
      IF((R.GT.144..OR.BB(N,1).LT.0.00015).AND.JSW.GE.0) IERFLG=1
C     IF BELOW EARTHS SURFACE SET FLAG NEGATIVE
      IF(RSV(N).LT.1.02) IERFLG=-1
      RETURN
      END
```

```
      SUBROUTINE INTERP(BB,CC,D,E,J)
C
C     PURPOSE
C        INTERPOLATION ROUTINE
C
C     METHOD
C        GIVEN A SET OF THREE X,Y POINT PAIRS, INTERP FINDS THE SOLUTION
C        TO THE THREE LINEAR EQUATIONS EXPRESSING THE LOGARITHM OF THE
C        DEPENDENT VARIABLE Y AS A SECOND ORDER POLYNOMIAL OF THE
C        INDEPENDENT VARIABLE X. (LOG Y = A*X**2 +B*X +C)
C        USING THE BINOMIAL FORMULA, X CAN THEN BE EVALUATED AT A
C        SPECIFIED VALUE OF Y1
C        X = (-B +- SQRT(B**2-4*A*(C-LOG(Y1))))/(2*A)
C
C     INPUT -- ARGUMENT LIST
C        BB      A REAL ARRAY CONTAINING THE THREE VALUES OF THE
C                DEPENDENT VARIABLE
C        CC      A REAL ARRAY CONTAINING THE THREE CORESPONDING VALUES
C                OF THE INDEPENDENT VARIABLE
C        J       A FLOW CONTROL VARIABLE
C                IF J IS LESS THAN 0
C                FIT THE POLYNOMIAL TO CC AND BB AND FIND THE MINIMUM
C                VALUE OF THE DEPENDENT VARIABLE
C                IF J IS GRATER THAN 0
C                USE THE BINOMIAL FORMULA TO TO FIND THE VALUE OF
C                THE INDEPENDENT VARIABLE WHEN THE DEPENDENT VARIABLE
C                HAS THE VALUE D.CHOOSE THE ROOT THAT IS CLOSEST TO CC(J)
C        D       WHEN J IS GREATER THAN ZERO, D IS USED FOR INPUT.
C                IT IS THE VALUE OF THE DEPENDENT VARIABLE WHERE THE
C                SOLUTION TO THE DEPENDENT VARIABLE IS WANTED
C
C     OUTPUT -- ARGUMENT LIST
C        D       WHEN J IS LESS THAN 0, D OUTPUTS THE VALUE OF THE
C                DEPENDENT VARIABLE WHERE THE FUNCTION IS A MINIMUM
C        E       WHEN J IS LESS THAN 0, E OUTPUTS THE VALUE OF THE
C                INDEPENDENT VARIABLE WHERE THE FUNCTION IS A MINIMUM
C                WHEN J IS GREATER THAN 0, E OUPUTS THE VALUE OF THE
C                INDEPENDENT VARIABLE WHERE THE FUNCTION HAS THE VALUE D
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     VARIABLES
C        X2,X3,Y1,Y2,Y3,DD  ARE USED BY THE LINEAR EQUATION SOLUTION
C                TO MINIMIZE COMPUTER TIME
C        A,B,C   THE THREE POLYNOMIAL COOEFICIENTS
C        DIS     B**2-4*A*C
C        SA,SB   THE TWO ROOTS OF THE POLYNOMIAL
C
      DIMENSION BB(3),CC(3)
      REAL*8 Y1,Y2,Y3,X2,X3,DD,A,B,C,DIS
C
C     SET UP THE INITIAL VARIABLES, MOVE THE ORIGIN OF THE INDEPENDENT
C     VARIABLE TO CC(1)
C
c      if(j.gt.0) then
      Y1=ALOG(BB(1))
      Y2=ALOG(BB(2))
      Y3=ALOG(BB(3))
c      else
```

```
c        y1=bb(1)
c        y2=bb(2)
c        y3=bb(3)
c        endif
       X2=CC(2)-CC(1)
       X3=CC(3)-CC(1)
C
C      SOLVE THE LINEAR EQUATIONS
       DD=(X3-X2)*X2*X3
       IF(DD.EQ.0)THEN
       IF(J.LT.0) THEN
         D=BB(2)
       ELSE
         E=CC(J)
       ENDIF
       RETURN
       ENDIF
       A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
       B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
C
C      IF J THE FLOW CONTROL VARIABLE IS LESS THAN ZERO BRANCH TO
C      MINIMUM EVALUATION ROUTINE
       IF(J.LT.0)GO TO 100
       C=Y1-DLOG(D)
       DIS=B**2-4.*A*C
C
C      IF DIS IS NEGATIVE NO SOLUTION EXIST, EXCHANGE DEPENDENT AND
C      INDEPENDENT VARIABLE ROLES AND TRY ANOTHER SOLUTION
       IF(DIS.LE.0.) GO TO 200
       DIS=DSQRT(DIS)
C
C      OBTAIN THE TWO ROOTS
       SA=(-B+DIS)/(2.*A)+CC(1)
       SB=(-B-DIS)/(2.*A)+CC(1)
       E=SA
C
C      FIND THE ROOT CLOSEST TO CC(J)
       IF(ABS(SB-CC(J)).LT.ABS(SA-CC(J)))E=SB
       RETURN
C
C      FIND THE VALUES AT THE MINIMUM
  100  X=-B/(2.*A)
       E=X+CC(1)
       XM=A*X**2+B*X+Y1
       D=EXP(XM)
c        d=xm
       RETURN
C
C      ALTERNATE INTERPOLATION SCHEME PLACED HERE AS A SAFEGUARD
C      AGAINST A STRANGE FIELD CONFIGURATION CAUSING AN IMAGINARY
C      SOLUTION (EXCHANGE THE ROLES OF DEPENDENT AND INDEPENDENT
C      VARIABLES)
C
  200  Y1=CC(1)
       Y2=CC(2)
       Y3=CC(3)
       X2=BB(2)-BB(1)
       X3=BB(3)-BB(1)
       DD=(X3-X2)*X2*X3
       IF(DD.EQ.0)THEN
```

```
E=CC(J)
RETURN
ENDIF
A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
DX=D-BB(1)
E=(A*DX+B)*Dx+Y1
RETURN
END
```

```
      SUBROUTINE MGLONG(X,S,SF,XMLONG,RMIN,RMAG)
C
C     PURPOSE
C        TO DETERMINE THE MAGNETIC LONGITUDE OF THE MINIMUM B LOCATION
C        OF THE MAGNETIC FIELD LINE
C
C     METHOD
C        GIVEN A LOCUS OF POSITIONS ALONG A FIELD LINE AS A FUNCTION
C        OF THE SCALAR DISTANCE ALONG THE FIELD LINE AND GIVEN THE
C        SCALAR DISTANCE WHERE THE FIELD IS A MINIMUM, THE ROUTINE
C        FINDS THE VECTOR POSITION OF THE MINIMUM.  IT THEN TRANSFORMS
C        THIS MINIMUM TO OFFSET DIPOLE COORDINATES AND CALCULATES
C        THE MAGNETIC LONGITUDE OF THE MINIMUM
C        NOTE*******THE CONSTANT ISWTCH IS SET BY A DATA STATEMENT,
C        IF IT IS SET TO ZERO XMLONG IS CALCULATED USING A CENTERED
C        DIPOLE COORDINATE SYSTEM WITH ZERO LONGITUDE AT 69 DEGREES
C        WEST GEOGRAPHIC.  IF ISWTCH IS SET NON-ZERO, AN OFFSET DIPOLE
C        COORDINATE SYSTEM IS USED WITH XMLONG=0 GOING THROUGH
C        GREENWHICH
C
C     INPUT -- ARGUMENT LIST
C        X       A REAL 2 DIMENSIONED ARRAY CONTAINING THE LOCUS OF
C                POINTS ALONG A FIELD LINE
C                X(1,1), X(1,2) AND X(1,3) ARE THE X, Y, Z VALUES
C                (RIGHT HANDED CARTESIAN COORDINATES) AT THE FIRST
C                POINT, X(2,1), X(2,2) AND X(2,3) THE SECOND LOCATION
C                AND X(3,1), X(3,2) AND X(3,3) ARE AT THE THIRD LOCATION
C                THE FIRST DIMENSION OF X MUST BE THE SAME AS THE
C                CALLING PROGRAMS DIMENSION - IN THIS CASE IT IS 100
C        S       A REAL ARRAY CONTAINING THE SCALAR DISTANCE ALONG THE
C                FIELD LINE IN EARTH RADII. S(1) IS THE SCALAR DISTANCE
C                TO THE X(1,1), X(1,2), X(1,3) POINT FROM THE START
C                OF THE INTEGRATION, S(2) IS THE DISTANCE TO X(2,1),...
C        SF      THE SCALAR DISTANCE TO THE MAGNETIC MINIMUM
C
C     OUTPUT -- ARGUMENT LIST
C        XMLONG THE MAGNETIC LONGITUDE (IN DEGREES) OF THE MINIMUM
C                OF THE MAGNETIC LINE OF FORCE
C                IF ISWTCH IS ZERO, THE ZERO OF MAGNETIC LONGITUDE IS
C                ALONG 69 DEG WEST GEOGRAPHIC
C                IF ISWTCH IS NOT ZERO, THE ZERO OF MAGNETIC LONGITUDE
C                IS THROUGH GREENWHICH
C
C     CONSTANTS
C        DX      THE 3 VECTOR COMPONENTS OF THE LOCATION OF THE OFFSET
C                DIPOLE IN EARTH RADII (GEOGRAPHIC CARTESIAN COORDS)
C        A22-A34 TRANSFORMATION MATRIX TO OFFSET DIPOLE COORDS. FIRST
C                ROTATE ABOUT THE GEOGRAPHIC Z AXIS, TO THE MERIDIAN
C                CONTAINING THE OFFSET DIPOLE,  THEN ABOUT THE NEW Y AXIS
C                TO THE LATITUDE CONTAINING THE OFFSET DIPOLE AND THEN
C                ABOUT THE NEW Z AXIS SUCH THAT THE ZERO OF LONGITUDE
C                PASSES THROUGH GREENWHICH
C        ISWTCH A FLOW CONTROL CONSTANT
C                IF SET TO ZERO BY THE DATA STATEMENT USE CENTERED DIPOLE
C                COORDINATES
C                IF SET NON-ZERO USE OFFSET DIPOLE COORDINATES
C        SIN D   SINE OF THE COLATITUDE OF THE CENTERED DIPOLE AXIS
C        COS D   COSINE OF THE COLATITUDE OF THE CENTERED DIPOLE AXIS
C        S69     SINE OF 69 DEGREES
C        C69     COSINE OF 69 DEGREES
```

```
C
C     TEMPORARY VARIABLES
C        XF,X1,X2,Y1,Y2,Y3,A,B,DD  THESE VARIABLE ARE USED IN THE
C               INTERPOLATION LOOP TO MINIMIZE THE NUMBER OF MEMORY
C               REFERENCES AND TO MINIMIZE THE NUMBER OF MULTIPIES
C        XT     A REAL ARRAY HOLDING THE LOCATION OF THE MINIMUM AND
C               LATER THE OFFSET MINIMUM OF THE FIELD LINE
C        XP,YP  THE POSITION OF THE MINIMUM IN OFFSET MAGNETIC CORDS.
      DIMENSION X(100,3),S(100),DX(3),XT(3),RMIN(3)
      DATA DX(1),DX(2),DX(3)/0.0576,-0.0321,-0.0184/
      DATA A22,A23,A24,A32,A33,A34/0.97056,0.23948,-0.02556,
     *-0.22969,0.95232,0.20082/
      DATA SIND,COSD,S69,C69/.2027872954,.9792228106,.9335804265,
     *.3583679495/
C
C
C     ********SET UP THE FLOW CONTROL SWITCH**********
C     COORDINATE SYSTEM DEFINITION USED. (SEE METHOD)
      DATA ISWTCH/1/
C
C     BEGIN QUADRATIC INTERPOLATION
      XF=SF-S(1)
      X2=S(2)-S(1)
      X3=S(3)-S(1)
      DD=(X3-X2)*X2*X3
C
C     INTERPOLATE EACH COMPONENT SEPERATELY
      DO 10 I=1,3
      Y1=X(1,I)
      Y2=X(2,I)
      Y3=X(3,I)
      A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
      B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
C
C     EVALUATE THE POSITION OF THE MINIMUM
      XT(I)=(A*XF+B)*XF+Y1
      RMIN(I)=XT(I)
 10   CONTINUE
      RMAG2=XT(1)**2+XT(2)**2+XT(3)**2
      RMAG=SQRT(RMAG2)
C
C     IF ISWTCH IS ZERO GO TO CENTERED DIPOLE DEFINITION
      IF (ISWTCH.EQ.0) GO TO 30
C
C     ADD IN THE DIPOLE OFFSET
      DO 20 I=1,3
 20   XT(I)=XT(I)+DX(I)
C
C     TRANSFORM TO OFFSET DIPOLE COORDINATES AND EVALUATE THE LONGITUDE
      XP=A22*XT(1)+A23*XT(2)+A24*XT(3)
      YP=A32*XT(1)+A33*XT(2)+A34*XT(3)
      GO TO 40
C
C     TRANSFORM TO CENTERED DIPOLE COORDINATES
 30   XP=(XT(1)*C69-XT(2)*S69)*COSD-XT(3)*SIND
      YP=XT(1)*S69+XT(2)*C69
C
C     CALCULATE MAGNETIC LONGITUDE
 40   XMLONG=ATAN2(YP,XP)*57.2957795
      IF(XMLONG.LT.0.) XMLONG=XMLONG+360.
```

```
RETURN
END
```

```
      SUBROUTINE HILTEL (B,XI,VL)
C     PURPOSE
C        CALCULATE THE L VALUE
C        THE ORIGINAL MCILWAIN L EXPANSION GIVEN BY THE OLD
C        SUBROUTINE CARMEL HAS BEEN REPLACED BY HILTONS SIMPLER
C        EXPANSION.  DIFFERENCES BETWEEN HILTONS AND MCILWAINS
C        EXPANSION ARE TYPICALLY LESS THAN .01 PERCENT.
C
C     METHOD
C        SEE J. HILTON, J. GEOPHYS. RES. 76, 6952 (1971)
C
C     INPUT -- CALLING SEQUENCE
C        B       THE MAGNETIC FIELD AT THE PARTICLE MIRROR POINT
C        XI      THE SECOND INVARIANT EVALUATED BETWEEN MIRROR POINTS
C                EXPRESSED IN UNITS OF EARTH RADII
C
C     OUTPUT -- CALLING SEQUENCE
C        VL      THE L VALUE
C
C     THE NEXT STATEMENT CONTAINS THE ORIGINAL McILWAIN MOMENT
C     DATA XM/.311653/
C     USE THE DIPOLE MOMENT CALCULATED FROM THE CURRENT FIELD MODEL
      COMMON /MOMENT/XM
      IF(XI.GT.1.0E-36) GO TO 10
      VL=(XM/B)**(1./3.)
      RETURN
 10   X=XI*(B/XM)**(1./3.)
      V=1.+X*(1.35047+X*(.465376+.0475455*X))
      VL=(V*XM/B)**(1./3.)
C     END COMPUTE L
      RETURN
      END
```

Appendix E

The Vector Potential Model

Subroutine AXYZ

This subroutine is the basic vector potential subroutine. The routine was developed in 1977 along with the 1977 Olson Pfitzer tilt dependent magnetic field model. The routine calculates the magnetic vector potential in units of nanotesla-Re everywhere inside the magnetosphere and inside of a sphere of radius 13 Re. The routine is tilt dependent. The routine is a series of polynomials plus polynomials times an exponential. The complexity of the function is such that if the coefficients of the ring and tail are added to coefficients describes in this listing, the functions have sufficient fidelity to describe the detailed structure due to the ring and tail current systems.

Calling sequence

XX(3) a 3 dimension input array that specifies the position in Cartesian solar magnetic coordinates. XX(3) along the north dipole axis, XX(1) is perpendicular to XX(3) and in the plane containing XX(3) and the sun-earth line and pointing in the direction of the sun, XX(2) completes the right handed coordinate system. The distance are given in unit of Re.

AT(3) a 3 dimensioned array that returns the vector components of the magnetic vector potential. The units are in nanotesla-Re.

COMMON/TILTIT/TILT TILT is an input variable that specifies the tilt of the earth's dipole axis. Zero tilt indicates that the dipole is perpendicular to the sun-earth line. Positive tilt is when the northern dipole is tipped toward the sun. This value must be set up before a call is made to routine AXYZ.

```
SUBROUTINE AXYZ(XX,AT)
C  This routine caluclates the Vector potential of the magnetopause
c  magnetospheric magnetic field during quiet conditions for any tilt.
C  XX(3) is a real*8 position in earth radii in solar magnetic coords
C  AT(3) is the real*8 vector potential in nanotesla-Re
      REAL*8 TILT
      REAL*8 D(44),E(64),F(44),DM(88),EM(128),FM(88),
     *AT(3),X(10),Y(10),Z(10),XX(3),TT(4),AA(3),
     *TILTL,XN,YN,ZN,R2,R
      INTEGER*2 ITD(44),ITE(64),ITF(44),
     *I,II,K
      COMMON/TILTIT/TILT
      DATA ITD /1,2,1,2,1,1,2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1,1,2,1,2,1,1,
     *2,1,1,1,2,1,2,1,1,2,2,1,2,1,1,1/
      DATA ITE /1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,
     *1,1,1,2,1,2,1,2,1,1,2,1,1,1,2,1,1,2,1,1,2,2,2,2,2,1,2,1,1,1,1,2,1,
     *1,1,2/
      DATA ITF /2,1,2,1,2,2,1,2,2,2,1,2,1,2,2,1,1,2,1,2,2,2,2,1,2,1,2,2,
     *1,2,2,2,1,2,1,2,2,1,1,2,1,2,2,2/
      DATA (DM(I),I=1,88)/
     *-.729348268D+00,-.126711994D-03,-.250256245D-02,-.929734000D-06,
     *-.651629108D-01,-.166931408D-04,-.429000833D-03,-.274123969D-07,
     *-.313618130D-02,-.569972419D-05, .101581273D-02, .210875857D-05,
     * .100224170D-04, .447416904D-08, .157165005D-04, .371435608D-07,
     *-.302681932D-03, .526556403D-06, .139932388D-03, .163943276D-06,
     * .236910507D-05, .404562068D-09,-.174908288D-02, .111043613D-05,
     * .395551400D-05, .196463764D-07,-.172010105D-04, .104333857D-06,
     * .328361431D-05,-.381150320D-08, .188742505D-05, .796065264D-08,
     * .344430885D-06, .133982119D-08, .245745657D-03, .220385117D-06,
     * .245904885D-05, .986230049D-09, .183409240D-04, .265129601D-07,
     *-.212837026D-05,-.872199184D-08, .172169713D-04, .294076230D-08,
     *44*0./
      DATA (EM(I),I=1,128)/
     * .539534465D+01,-.917246729D-03,-.202212227D-02,-.129650634D-05,
     * .848876852D+00, .710329502D-04, .305008723D-02, .120551460D-05,
     *-.598376603D-01,-.165177282D-06,-.334958332D-02, .472213553D-05,
     * .422206531D-04, .173170587D-07, .160966776D-03, .208088996D-08,
     * .201674318D-02, .869926096D-05,-.170424338D-02,-.280163907D-05,
     *-.741748071D-05,-.919757909D-08,-.426006943D-05,-.533888339D-07,
     * .459399443D-01, .163072391D-04, .445957685D-03, .511926130D-08,
     * .562030059D-03,-.263083865D-06,-.400241568D-04,-.228038044D-06,
     *-.265641610D-05,-.173364107D-09,-.111586849D-03, .487350690D-07,
     * .558513796D-06,-.355505259D-09,-.173441325D-04,-.139197938D-08,
     * .149384635D-05,-.184142610D-08,-.565550702D-03,-.127131636D-05,
     *-.377826457D-05,-.241832155D-07, .395029200D-04,-.150796074D-06,
     * .141863427D-04, .157547770D-07,-.894741777D-05, .441400372D-07,
     * .869472657D-05,-.170170836D-07,-.123151650D-06,-.675272345D-10,
     *-.549566250D-04,-.134417266D-07, .538339684D-05, .119398067D-07,
     *-.105866429D-03,-.224420771D-06,-.298911521D-05,-.432303236D-09,
     *64*0./
      DATA (FM(I),I=1,88)/
     * .452063859D-02, .309321235D-06, .658118267D-01, .922543691D-05,
     * .221930272D-03,-.269376799D-06, .434366116D-02,-.169477296D-05,
     * .264268049D-03,-.496539062D-07,-.383052388D-04,-.818745994D-08,
```

```
     *-.139054930D-03,-.902753036D-07,-.122044254D-05, .305995114D-09,
     * .299405601D-05,-.823980252D-08,-.137081926D-06, .686745107D-09,
     *-.449407954D-05, .516272489D-08,-.108807156D-03,-.225109712D-07,
     *-.387313966D-03,-.181461377D-06,-.257814468D-05, .362172735D-09,
     * .107887529D-05,-.817801019D-10, .327270756D-04,-.213768490D-06,
     * .182395159D-04,-.467691071D-07,-.425729821D-05, .457564728D-08,
     *-.507417182D-04, .234472682D-07,-.828838728D-06,-.222432842D-09,
     * .118487595D-06, .259558968D-10, .445023379D-06, .345792292D-09,
     *44*0./
      DATA TILTL/99.D+0/
      IF(TILT.EQ.TILTL)GO TO 20
      TILTL=TILT
      TT(1)=1.
      TT(2)=TILT
      TT(3)=TILT*TILT
      TT(4)=TT(3)*TILT
      DO 10 I=1,64
      II=(I-1)*2+1
      K=ITD(I)
      if(i.le.44)D(I)=(10.*DM(II))*TT(K)+(10.*DM(II+1))*TT(K+2)
      K=ITE(I)
      E(I)=(10.*EM(II))*TT(K)+(10.*EM(II+1))*TT(K+2)
      K=ITF(I)
 10   if(i.le.44)F(I)=(10.*FM(II))*TT(K)+(10.*FM(II+1))*TT(K+2)
 20   CONTINUE
      XN=XX(1)
      YN=XX(2)
      ZN=XX(3)
      R2=XN**2+YN**2+ZN**2
      R=SQRT(R2)
      DO 1 I=1,7
      X(I)=XN
      Y(I)=YN
      Z(I)=ZN
      XN=XN*XX(1)
      YN=YN*XX(2)
      ZN=ZN*XX(3)
  1   CONTINUE
      AA(1)=+D( 1)*Y( 1)+D( 2)*Y( 1)*Z( 1)+D( 3)*X( 1)*Y( 1)+D( 4)*X( 1)
     **Y( 1)*Z( 1)+D( 5)*Y( 1)*Z( 2)+D( 6)*Y( 3)+D( 7)*Y( 3)*Z( 1)+D( 8)
     **Y( 3)*Z( 2)+D( 9)*X( 1)*Y( 1)*Z( 2)+D(10)*X( 1)*Y( 3)+D(11)*X( 1)
     **Y( 3)*Z( 1)+D(12)*X( 2)*Y( 1)+D(13)*X( 2)*Y( 1)*Z( 1)+D(14)*X( 2)
     **Y( 1)*Z( 2)+D(15)*X( 2)*Y( 3)+D(16)*Y( 1)*Z( 3)+D(17)*X( 1)*Y( 1)
     **Z( 3)+D(18)*X( 3)*Y( 1)+D(19)*X( 3)*Y( 1)*Z( 1)+D(20)*Y( 1)*Z( 4)
     *+D(21)*Y( 5)+D(22)*X( 4)*Y( 1)
      AA(1)=AA(1)+(0.0 +D(23)*Y( 1)+D(24)*Y( 1)*Z( 1)
     *+D(25)*X( 1)*Y( 1)+D(26)*X( 1)*Y( 1)*Z( 1)+D(27)*Y( 1)*Z( 2)+D(28)
     **Y( 3)+D(29)*Y( 3)*Z( 1)+D(30)*Y( 3)*Z( 2)+D(31)*X( 1)*Y( 1)*Z( 2)
     *+D(32)*X( 1)*Y( 3)+D(33)*X( 1)*Y( 3)*Z( 1)+D(34)*X( 2)*Y( 1)+D(35)
     **X( 2)*Y( 1)*Z( 1)+D(36)*X( 2)*Y( 1)*Z( 2)+D(37)*X( 2)*Y( 3)+D(38)
     **Y( 1)*Z( 3)+D(39)*X( 1)*Y( 1)*Z( 3)+D(40)*X( 3)*Y( 1)+D(41)*X( 3)
     **Y( 1)*Z( 1)+D(42)*Y( 1)*Z( 4)+D(43)*Y( 5)+D(44)*X( 4)*Y( 1))*EXP
     *( -.06*R2)
      AA(2)=+E( 1)+E( 2)*Z( 1)+E( 3)*X( 1)+E( 4)*X( 1)*Z( 1)+E( 5)*Z( 2)
```

```
*+E( 6)*Y( 2)+E( 7)*Y( 2)*Z( 1)+E( 8)*Y( 2)*Z( 2)+E( 9)*X( 1)*Z( 2)
*+E(10)*X( 1)*Y( 2)+E(11)*X( 1)*Y( 2)*Z( 1)+E(12)*X( 1)*Y( 2)*Z( 2)
*+E(13)*X( 2)+E(14)*X( 2)*Z( 1)+E(15)*X( 2)*Z( 2)+E(16)*X( 2)*Y( 2)
*+E(17)*X( 2)*Y( 2)*Z( 1)+E(18)*Z( 3)+E(19)*Y( 2)*Z( 3)+E(20)*X( 1)
**Z( 3)+E(21)*X( 2)*Z( 3)+E(22)*X( 3)+E(23)*X( 3)*Z( 1)+E(24)*X( 3)
**Z( 2)+E(25)*X( 3)*Y( 2)+E(26)*Z( 4)+E(27)*Y( 4)+E(28)*Y( 4)*Z( 1)
*+E(29)*X( 1)*Z( 4)+E(30)*X( 1)*Y( 4)+E(31)*X( 4)+E(32)*X( 4)*Z( 1)
 AA(2)=AA(2)
*+(0.0 +E(33)+E(34)*Z( 1)+E(35)*X( 1)+E(36)*X( 1)*Z( 1)+E(37)*Z( 2)
*+E(38)*Y( 2)+E(39)*Y( 2)*Z( 1)+E(40)*Y( 2)*Z( 2)+E(41)*X( 1)*Z( 2)
*+E(42)*X( 1)*Y( 2)+E(43)*X( 1)*Y( 2)*Z( 1)+E(44)*X( 1)*Y( 2)*Z( 2)
*+E(45)*X( 2)+E(46)*X( 2)*Z( 1)+E(47)*X( 2)*Z( 2)+E(48)*X( 2)*Y( 2)
*+E(49)*X( 2)*Y( 2)*Z( 1)+E(50)*Z( 3)+E(51)*Y( 2)*Z( 3)+E(52)*X( 1)
**Z( 3)+E(53)*X( 2)*Z( 3)+E(54)*X( 3)+E(55)*X( 3)*Z( 1)+E(56)*X( 3)
**Z( 2)+E(57)*X( 3)*Y( 2)+E(58)*Z( 4)+E(59)*Y( 4)+E(60)*Y( 4)*Z( 1)
*+E(61)*X( 1)*Z( 4)+E(62)*X( 1)*Y( 4)+E(63)*X( 4)+E(64)*X( 4)*Z( 1)
*)*EXP ( -.06*R2)
 AA(3)=+F( 1)*Y( 1)+F( 2)*Y( 1)*Z( 1)+F( 3)*X( 1)*Y( 1)+F( 4)*X( 1)
**Y( 1)*Z( 1)+F( 5)*Y( 1)*Z( 2)+F( 6)*Y( 3)+F( 7)*Y( 3)*Z( 1)+F( 8)
**Y( 3)*Z( 2)+F( 9)*X( 1)*Y( 1)*Z( 2)+F(10)*X( 1)*Y( 3)+F(11)*X( 1)
**Y( 3)*Z( 1)+F(12)*X( 2)*Y( 1)+F(13)*X( 2)*Y( 1)*Z( 1)+F(14)*X( 2)
**Y( 1)*Z( 2)+F(15)*X( 2)*Y( 3)+F(16)*Y( 1)*Z( 3)+F(17)*X( 1)*Y( 1)
**Z( 3)+F(18)*X( 3)*Y( 1)+F(19)*X( 3)*Y( 1)*Z( 1)+F(20)*Y( 1)*Z( 4)
*+F(21)*Y( 5)+F(22)*X( 4)*Y( 1)
 AA(3)=AA(3)+(0.0 +F(23)*Y( 1)+F(24)*Y( 1)*Z( 1)
*+F(25)*X( 1)*Y( 1)+F(26)*X( 1)*Y( 1)*Z( 1)+F(27)*Y( 1)*Z( 2)+F(28)
**Y( 3)+F(29)*Y( 3)*Z( 1)+F(30)*Y( 3)*Z( 2)+F(31)*X( 1)*Y( 1)*Z( 2)
*+F(32)*X( 1)*Y( 3)+F(33)*X( 1)*Y( 3)*Z( 1)+F(34)*X( 2)*Y( 1)+F(35)
**X( 2)*Y( 1)*Z( 1)+F(36)*X( 2)*Y( 1)*Z( 2)+F(37)*X( 2)*Y( 3)+F(38)
**Y( 1)*Z( 3)+F(39)*X( 1)*Y( 1)*Z( 3)+F(40)*X( 3)*Y( 1)+F(41)*X( 3)
**Y( 1)*Z( 1)+F(42)*Y( 1)*Z( 4)+F(43)*Y( 5)+F(44)*X( 4)*Y( 1))*EXP
*( -.06*R2)
 AT(1)=AA(1)
 AT(2)=AA(2)
 AT(3)=AA(3)
 RETURN
 END
```

Appendix F

Time Dependent Routines for March Event

F.1 Subroutine AXYZDN

Subroutine AXYZDN is the disturbed condition vector potential model. The routine calls subroutine SMAG which provides the correct scaling parameters for this routine. The subroutine provides the correct disturbed time vector potential at time t providing subroutine SMAG provides the correct scaling parameters. The scaling parameters that are used are STRMAG, the strength of the magnetopause currents, and SCL the size scaling parameter. These parameters are discussed in section 5.6

F.1.1 Calling Sequence

XX(3) a 3 dimension input array that specifies the position in Cartesian solar magnetic coordinates. XX(3) along the north dipole axis, XX(1) is perpendicular to XX(3) and in the plane containing XX(3) and the sun-earth line and pointing in the direction of the sun, XX(2) completes the right handed coordinate system. The distance are given in unit of Re.

AT(3) a 3 dimensioned array that returns the vector components of the disturbed condition magnetic vector potential. The units are in nanotesla-Re

T The time during the event. The time must be in units of seconds. The time, T, is passed through to SMAG, where SMAG must use it to determine the scaling parameters.

COMMON/TILTIT/TILT TILT is also an input variable. It specifies the tilt of the earth's dipole axis. Zero tilt indicates that the dipole is perpendicular to the sun-earth line. Positive tilt is when the northern dipole is tipped toward the sun. This value must be set up before a call is made to routine AXYZDN.

F.1.2 Subroutine Listing - AXYZDN

```
      subroutine axyzdyn(x,a,t)
C  This is the disturbed condition Vector potential.  I uses the scaling
C  algorithms developed in 1982 (JGR Aug. 82 p5943)
C  It requires that subroutine SMAG return the magnetopause current
C  and magnetopause scale size as a function of time
C  X(3) is the position in Re in solar magnetic coord
C  A(3) is the Vector potential in nanotesla-Re
C  T is the time in seconds
      REAL*8 a(3),x(3),xx(3),aa(3),STRMAG,SCL,T
      INTEGER*2 I
      call smag(strmag,scl,t)
```

```
      do 110 i=1,3
110   xx(i)=x(i)*scl
      call axyz(xx,aa)
      do 120 i=1,3
120   a(i)=aa(i)*(SCL**2)
      return
      end
```

F.2 Subroutine CURLA

This subroutine calculates the curl of the quiet time vector potential. It produces the a quiet time magnetic field model, that except for the precision in the fit and numerical derivatives will be very similar to the Olson Pfitzer 1977 tilt dependent model.

F.2.1 Calling Sequence

XX(3) a 3 dimension input array that specifies the position in Cartesian solar magnetic coordinates. XX(3) along the north dipole axis, XX(1) is perpendicular to XX(3) and in the plane containing XX(3) and the sun-earth line and pointing in the direction of the sun, XX(2) completes the right handed coordinate system. The distance are given in unit of Re.

BBB(3) a 3 dimensioned array that returns the vector components of the quiet time magnetic field. The units are in nanotesla.

COMMON/TILTIT/TILT TILT is also an input variable. It specifies the tilt of the earth's dipole axis. Zero tilt indicates that the dipole is perpendicular to the sun-earth line. Positive tilt is when the northern dipole is tipped toward the sun. This value must be set up before a call is made to routine CURLA.

F.2.2 Program Listing -- Subroutine CURLA

```
      SUBROUTINE CURLA(XX,BBB)
C  This subroutine calculates the numerical CURL of the Vector
C  potential and thus calculates the quiet time magnetic field.
C  It calls AXYZ and thus returns the value of B in nanotesla
C  XX is the Real*8 value of the position in Earth radii
C  BBB is the Real*8 value of the magnetic field in nanotesla
C  DEL is a step size parameter for the numercial CURL
      REAL*8 X(3),B(3),BB(3,3),BBB(3),xx(3)
     *,DEL
      INTEGER*2 I,J,K
      DATA DEL/0.0001/
      do 1 i=1,3
1     x(i)=xx(i)
      CALL AXYZ(X,B)
      DO 10 I=1,3
      X(I)=X(I)+DEL
      CALL AXYZ(X,BB(1,I))
10    X(I)=X(I)-DEL
      DO 20 I=1,3
      J=I+2
      J=J-(J-1)/3*3
      K=I+1
```

```
      K=K-(K-1)/3*3
20    BBB(I)=(BB(J,K)-B(J)-BB(K,J)+B(K))/DEL
      RETURN
      END
```

F.3 Subroutine DYNB

This subroutine calculates the disturbed of dynamic magnetic field values. It produces the a disturbed time magnetic field model using the curl of the vector potential.

F.3.1 Calling Sequence

XX(3) a 3 dimension input array that specifies the position in Cartesian solar magnetic coordinates. XX(3) along the north dipole axis, XX(1) is perpendicular to XX(3) and in the plane containing xx(3) and the sun-earth line and pointing in the direction of the sun, XX(2) completes the right handed coordinate system. The distance are given in unit of Re.

B(3) a 3 dimensioned array that returns the vector components of the disturbed time magnetic field. The units are in nanotesla.

BMAG returns the magnitude of the disturbed time magnetic field in units of nanotesla

T an input variable that gives the time during the event. This time, T, must be in units of seconds. The time, T, is passed through to SMAG, where SMAG must use the time to determine the scaling parameters.

COMMON/TILTIT/TILT TILT is also an input variable. It specifies the tilt of the earth's dipole axis. Zero tilt indicates that the dipole is perpendicular to the sun-earth line. Positive tilt is when the northern dipole is tipped toward the sun. This value must be set up before a call is made to routine DYNB.

F.3.2 Program Listing -- Subroutine DYNB

```
      subroutine dynb(x,b,bmag,t)
C   This is the disturbed condition magnetospheric magnetic field model
C   It determines the magnetopause magnetic field as a function of time
C   It requires that subroutine SMAG determines the magnetospheric
C   current strength and magnetospheric scaling parameter as a function
C   of time
C   X(3) is the position in Re in solar magnetic coord
C   B(3) is the mangetic field in nanotesla
C   BMAG is the mangetic field magnitude
C   T is the time in seconds
       REAL*8 x(3),xx(3),b(3),bb(3),BMAG,T,STRMAG,SCL
       INTEGER*2 I
       call smag(strmag,scl,t)
       do 210 i=1,3
210    xx(i)=x(i)*scl
       call curlA(xx,bb)
       do 220 i=1,3
220    b(i)=bb(i)*strmag
```

```
bmag=dsqrt(b(1)**2+b(2)**2+b(3)**2)
return
end
```

F.4 Subroutine EFIELD

This subroutine calculates the induction electric field from the changing Chapman Ferraro currents. It produces an induction electric field as a function of time when the Chapman-Ferraro currents are changing in response to a changing solar wind pressure.

F.4.1 Calling Sequence

XX(3) a 3 dimension input array that specifies the position in Cartesian solar magnetic coordinates. XX(3) along the north dipole axis, XX(1) is perpendicular to XX(3) and in the plane containing XX(3) and the sun-earth line and pointing in the direction of the sun, XX(2) completes the right handed coordinate system. The distance are given in units of meters.

E(3) a 3 dimensioned array that returns the vector components of the disturbed time magnetic field. The units are in Volts/meter.

EMAG returns the magnitude of the disturbed time magnetic field in units of Volts/meter

T an input variable that gives the time during the event. The time must be in units of seconds. The time, T, is passed through to SMAG, where SMAG must use it to determine the scaling parameters.

COMMON/TILTIT/TILT TILT is a variable used by the vector potential program. This routine sets the value of TILT to 0. The tilt is the tilt of the earth's dipole axis. Zero tilt indicates that the dipole is perpendicular to the sun-earth line. Positive tilt is when the northern dipole is tipped toward the sun. This value must be set up before a call is made to routine EFIELD.

F.4.2 Program Listing -- Subroutine EFIELD

```
      subroutine efield(xx,E,emag,t)
C  This routine calculates the induction electric field.  It must be used
C  in MKS units.
C  XX(3) is the position is entered in meters (solar magnetic coords)
C  E(3) returns the vector induction magnetic field in Volts/meter
C      E = negative of the time derivative of the vector potential
C  Emag is the magnitude of the induction electric field.
C  T is the time in seconds.  Subroutine SMAG must be properly set up
C  to give the magnetospheric boundary parameters as a function of the
C  time in seconds
      REAL*8 X(3),XX(3),a1(3),a2(3),E(3)
      REAL*8 EMAG,T,TILT,T1,T2,DELT,CON
      INTEGER*2 I
      COMMON/TILTIT/TILT
```

```
      data con/6.371D-3/,delt/0.0001/
      do 10 i=1,3
10    x(i)=xx(i)/6.371D+6
      tilt=0
      t1=t
      t2=t1+delt
      call axyzdyn(x,a1,t1)
      call axyzdyn(x,a2,t2)
      E(1)=-(a2(1)-a1(1))/delt*con
      E(2)=-(a2(2)-a1(2))/delt*con
      E(3)=-(a2(3)-a1(3))/delt*con
      emag=dsqrt(E(1)**2+E(2)**2+E(3)**2)
      return
      END
```

F.5 Subroutine SMAG

This routine is the time dependent driver routine that must be modified by the user to give the time dependent magnetopause scaling factors. The routine must calculate the strength of the magnetopause currents, and the scale size of the magnetosphere as a function of the time, t. Time must be in units of seconds. The scale factor SCL is given by

$$SCL = 10.5/R_S$$

The magnetopause current strength factor STRMAG is given by

$$STRMAG = \left[\frac{10.5}{R_S}\right]^3$$

Where R_S is the standoff distance. This srength vactor is used to scale the magnetic field. The Vecot potential is scaled by a power of 2 instead of a power of 3.

F.5.1 Calling Sequence

T is the time in units of seconds. It is passed through by the various field routines. This routine converts time to the time dependent scale factors and magnetopause current strength factor.

STRMAG returns the strength of the magnetopause current at time t

SCL is the time dependent scale factor that scales the positions with respect to the size of the magnetopause.

F.5.2 Program listing -- Subroutine SMAG

```
      subroutine smag(strmag,scl,t)
C  This subroutine gives the standoff distance and scaling parameters
C  as a funtion of the time in seconds
      REAL*8 STRMAG,SCL,T,STDOFF
C
C  This is an example of a linear change in the standoff distance at
C    .3 Re/second
C
      goto 800
      STDOFF=10.5-.3*T
      SCL=10.5/STDOFF
      STRMAG=SCL**3
      return
C
C  This is an example of a different form for the change in magnetopause
C  configuration.  It tries to match the CRRES dB/dt observation for
C  rev 587 during the first 30 seconds of the event.
C
```

```
800   if (t.le.0)then
        strmag=1
      else
        strmag=1+9*((t/30)**1.3)
      endif
      scl=strmag**0.3333333
      stdoff=10.5/scl
      end
```

Appendix G

Lorentz Force Integration Program

This section briefly describes a Runge Kutta trajectory integration program. It is included in this report for completeness since it was used to verify proton acceleration. It will permit any user to reproduce the values calculated in this document. The Runge-Kutta techniques used in this program are derived from a cosmic ray cut-off code that is over 20 years old. The code which was initially written to integrate the path as a function on position. The code used variable size distance steps and was written to hold the energy of a particle fixed. The code was modified to include the electric field and the distance stepping algorithm was changed to a time step code. The code has variable step size. The step size depends on the Larmor radius of the particle trajectory and the drift rate of the electric field force. The code uses MKS units throughout.

Since the program was not developed for general use, no attempt is made here to describe each of the routines in detail. The routines contain a substantial number of comment cards. This along with the overall simplicity of the code should permit most user to successfully reproduce the work in this document.

```
PROGRAM TRAJCHK
C  This is a driver program that sets up a call to the trajectory program
c
      real*8 X(6),v,tilt,xmag,t
      real*4 r,xlong,th,ph,w,an1,an2,an
      integer*2 i
      common/tiltit/tilt
C  Set tilt angle to zero and request starting coords,  Enter distance in Re
c  Angle from noon is + toward +y or dusk, pitch is angle with repect to z=0
c  plane, azimuth is + toward +x
      tilt=0
      print *,'Enter R,Angle from noon,Azimuth,Pitch,W'
1005  read *,R,xlong,th,ph,W
c  Tape 7 writes a file of the results, this copy is set up to inegrate
c  backward in time.  To change to forward a sign must be changed in 3
c  locations these are flagged by C$$$$$$$$$$$$$$$$
      write(7,*)' Backward'
      write(7,*)r,xlong,th,ph,w
      an1=th*3.14159/180.
      an2=ph*3.14159/180
      call veloc(W,v)
      print *,w,v
      an=xlong*3.14159/180.
C  Set up intial posistion X(1) thru X(3) hold particle position in meters
c  X(4) thru x(6) hold particles intitial velocity in meters/second
C
      X(1)=r*6.371e+6*cos(an)
      X(2)=r*6.371E+6*sin(an)
      X(3)=0
      X(4)=cos(an1)*sin(an2)
      X(5)=sin(an1)*sin(an2)
      X(6)=cos(an2)
      xmag=dsqrt(x(4)**2+x(5)**2+x(6)**2)
      do 5 i=4,6
5     x(i)=x(i)/xmag*V
      t=30.
      CALL TRAJPRO(X,t)
      END

      SUBROUTINE VELOC(W,V)
C  Determine the initial velocity of the proton given its energy in MeV
c  W is the energy in MeV
c  V is the velocity in meter/second
      REAL*8 V,V2C2
      V2C2=W*(W+2*931)/(W+931)**2
      V=3.0E+8*DSQRT(V2C2)
      RETURN
      END

      SUBROUTINE TRAJPRO(X,tt)
C  Calculate one complete particle trajectory stating at position xx
C  and time t.  XX is in meters and time is in seconds
      integer*4 n,number
      real*8 X(6),S(6),RR(6),Q(6),t,tt
```

```
      real*8 dxdt,xx,bb,bmag,e,emag,eb,RTPF,P29,OP7,DS,DT,DV,CON,
     *P5DS,P29DS,OP7DS,DIST
      integer*2 numb,i
      real*4 RRR,angle,en,xxx,xxy,xxz,xsv
      COMMON/SAD/DXDT(6),XX(6),BB(3),BMAG,E(3),EMAG,EB,t
      common/plotit/xsv(3,5000),number
      t=tt
      EB=1.
      numb=300
      number=0
      N=0
      DIST=0
      CON=.02
      DO 10 I=1,6
      XX(I)=X(I)
10    Q(I)=0
      RTPF=DSQRT(.5D0)
      P29=1.D0-RTPF
      OP7=1.D0+RTPF
      CALL EBFORCE
      EB=EMAG/BMAG
C
C  Main inegration loop  -- first set up variable step size
C  This uses Gill's method of Runge-Kutta
50    DS=CON*6.57E-8/BMAG
      DV=0.03*SQRT(XX(4)**2+XX(5)**2)
      if (eb.ne.0) then
      DT=1.05E-8/EMAG*DV
      DS=AMIN1(DS,DT)
      endif
      P5DS = .5*DS
      P29DS=P29*DS
      OP7DS=OP7*DS
C
C   GILL'S NUMERICAL INTEGRATION ROUTINE
C
      DO 60 I=1,6
      S(I) = P5DS*DXDT(I)
      RR(I)=S(I)-Q(I)
      XX(I)=XX(I)+RR(I)
   60 Q(I)=Q(I)+3.*RR(I)-S(I)
      CALL EBFORCE
      DO 61 I=1,6
      S(I) = P29DS*DXDT(I)
      RR(I)=S(I)-P29*Q(I)
      XX(I)=XX(I)+RR(I)
   61 Q(I)=Q(I)+3.*RR(I)-S(I)
      CALL EBFORCE
      DO 62 I=1,6
      S(I) = OP7DS*DXDT(I)
      RR(I)=S(I)-OP7*Q(I)
      XX(I)=XX(I)+RR(I)
   62 Q(I)=Q(I)+3.*RR(I)-S(I)
      CALL EBFORCE
```

```
      DO 63 I=1,6
      S(I) = P5DS*DXDT(I)
      RR(I)=(S(I)-Q(I))/3.
      XX(I)=XX(I)+RR(I)
   63 Q(I)=Q(I)+3.*RR(I)-S(I)
      N=N+1
      DIST = DIST + DS
C$$$$$$$$$$ to change to forward in time change sign to  +ds
      t=t-ds
      CALL EBFORCE
      RRR=DSQRT(XX(1)**2+XX(2)**2+xx(3)**2)/6.371E+6
C
C  Every so often print information on progress to the screen
      IF(N/100*100.EQ.N) then
      angle=datan2(xx(2),xx(1))*180./3.14159
      EN=(XX(4)*XX(4)+XX(5)*XX(5)+xx(6)**2)/1.912E+14
      WRITE(*,90)N,t,rrr,emag,angle,en
      endif
90    FORMAT(I7,f7.2,2f10.5,2f10.3)
C
C  Write stuff to a file every NUMB steps
      IF(N/NUMB*NUMB.EQ.N)THEN
      XXX=XX(1)/6.371E+6
      XXY=XX(2)/6.371E+6
      xxz=xx(3)/6.371e+6
      EN=(XX(4)*XX(4)+XX(5)*XX(5)+xx(6)**2)/1.912E+14
      angle=datan2(xx(2),xx(1))*180./3.14159
      WRITE (7,101) t,RRR,XXX,XXY,xxz,EN,emag,angle
      ENDIF
101   FORMAT(8f11.3)
100   FORMAT(I5,4E15.8,/,20X,3E15.8)
C  check to see if we are still in magnetosphere and that time is
C  still valid.
C  This exit condition must be changed for forward in time integration
C$$$$$$$$$$$$$$$$$
      IF (XXX.GT.-13.0 .and. rrr.lt.5
     *.and.t.gt.0) GOTO 50
      RETURN
      END

      SUBROUTINE EBFORCE
C  This routine calculates the Lorentz Force on a particle
C  DXDT(1),(2),(3) are the drivative of the position
C  DXDT(4),(5),(6) are the derivative of the velocity
C  all equations use MKS units
C
      REAL*8 DXDT,XX,BB,BMAG,E,EMAG,EB,t,dcon
      COMMON/SAD/DXDT(6),XX(6),BB(3),BMAG,E(3),EMAG,EB,t
C$$$$$$$$$$  to change to forward in time make constant +
      DATA DCON/-9.5D+7/
      DXDT(1)=XX(4)
      DXDT(2)=XX(5)
      DXDT(3)=XX(6)
      CALL BFIELD(XX,BB,BMAG,t)
```

```
      call efield(xx,e,emag,t)
C$$$$$$$$$$$$ to change to forward in time make change to +E in next 3 lines
20    DXDT(4)=DCON*(-E(1)+XX(5)*BB(3)-XX(6)*BB(2))
      DXDT(5)=DCON*(-E(2)+XX(6)*BB(1)-XX(4)*BB(3))
      DXDT(6)=DCON*(-E(3)+XX(4)*BB(2)-XX(5)*BB(1))
      RETURN
      END

      SUBROUTINE BFIELD(X,B,BMAG,t)
C  This routine combines a dipole field with a dynamic external field
C  It returns the magnetic field in MKS units (tesla)
C  XX the input position is in MKS unites (meters)
      REAL*8 X,B,BMAG,A,R,R2,t,cc,xx,btemp,bb
      integer*2 i
      DIMENSION X(1),B(1),xx(3),bb(3)
      DATA A/-8.1D+15/,cc/1.0D-9/
      do 300 i=1,3
300   xx(i)=x(i)/6.371e+6
      call dynb(xx,bb,btemp,t)
      R2=X(1)**2+X(2)**2+X(3)**2
      R=DSQRT(R2)
C  Convert external field to tesla and add dipole
      b(1)=3.d+0*x(1)*x(3)*a/(r2*r2*r)+bb(1)*cc
      b(2)=3.d+0*x(2)*x(3)*a/(r2*r2*r)+bb(2)*cc
      B(3)=(3.d+0*x(3)**2-r2)*A/(r2*R2*R)+bb(3)*cc
      BMAG=DSQRT(B(1)**2+B(2)**2+B(3)**2)
      RETURN
      END
```